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Investigation of Heat Transfer Characteristics of Spherical Copper and Alumina Nanoparticles in Water and Ethylene glycol Based Fluids

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ABSTRACT
A study of heat transfer rate of spherical copper and alumina nanoparticles in water and ethylene glycol based fluids was carried out. A modified thermal conductivity model in conjunction with steady state momentum and energy equations in spherical coordinates were put into dimensionless form and solutions used to determine the skin friction, heat transfer coefficient and thermal conductivity as well as viscosity. The modified model incorporates Brownian motion and varied sphericity to observe the effect of temperature and other material parameters on the velocity and temperature profiles of the fluid. Using numerical values, it was shown that the nanoparticle volume fractions, the diameter and Prandtl number, not only enhanced the thermal conductivity of nanofluids but also the velocity and temperature profiles. It was also observed that the Brownian motion which is temperature dependent was actually a weak factor in enhancement of thermal conductivity. The effect of other parameters as well as calculation of mass flux and mean temperature was determined.

Keywords: Heat transfer rate, Spherical coordinates, Copper, Alumina, Water, Ethylene glycol, Nanofluids.

1. INTRODUCTION

Generally, nanofluids are formed when some quantity of nanoparticles and base fluids are mixed. The nanoparticles has an average diameter between 1nm to 100nm.The nanoparticles are mainly oxides, metals and carbides while the common base fluids are water, ethylene glycol and other traditional heat transfer fluids. Nanofluids exhibit enhanced properties when compared to conventional heat transfer fluids and other metallic nanoparticles. The larger relative surface area of nanoparticles, compared to those of conventional particles, appears to justify significantly the enhanced heat transfer capabilities and also should improve the stability of the suspensions. Also, nanofluids enhanced abrasion-related properties as compared to the conventional solid/liquid mixtures. Successful use of nanofluids will support the component miniaturization by enabling the design and production of smaller and lighter heat exchanger systems. In his pioneer work, Choi [1], described the mixture as nanofluid with increased thermal conductivity of the base fluids and their convective heat transfer rate. The mixture or nanofluid when compared to the base fluid, it is observed that changes occur in viscosity, thermal conductivity and density. Nanofluids can therefore be described as new class of enhanced heat transfer fluids formed as a result of dispersion of nanometer-size particles in base fluids. Of the basic physical properties of nanofluids, thermal conductivity is the most important due to its application in heat transfer. The enhanced heat transfer was established because; nanofluid thermal conductivity is a function of nanoparticle size, shape, volume fraction and spatial distribution. Others are base fluid type, temperature and pH value (Yu et al [2], Eapen et al [3]). Nanofluids uses are abounded in various electronic equipment, energy production, power generation and air conditioning and production. Serious study in convective heat transfer using suspensions of nanometer-size particles in a chosen base fluid started only over the past decade. The study is either experimental or theoretical and a combination of both. Keblinski et al.[4], in a study, outline and examined the properties of nanofluids and future challenges. In spite of the several successes made, the research and development of nanofluids is seen as rapid but still hindered by several factors such as the lack of agreement between experimental and theoretical results, poor characterization of suspensions, and the lack of theoretical understanding of the mechanisms. This trend rather increased the study of naofluids and its applications. According to some proposed classical nanofluid models (Xie et al [5], Hamilton and Crosser [6], Jeffrey [7], Davis [8], Wang et al [9], Koo et al [10], Jang and Choi [11]), effective thermal conductivity of mixtures can be calculated and other parameters of the system also determined. However, results of some researches (Patel et al [12], Das et al [13], Xuan et al [14], Kumar et al [15], Bhattacharya et al [16], Putnam et al [17], Koo and Kleinstreuer [18 and 19]) strongly argued that the non inclusion of Brownian motion effect which is temperature dependent to the earlier stated models is a minus and therefore not a holistic approach to the determination of thermal conductivity enhancement, hence their studies. Surprisingly, Wang et al [20], in their study, declared that the effective thermal conductivity enhancement due to Brownian motion is not necessary. The implication is that the effect of temperature on the enhancement of thermal conductivity is very minimal and can be discarded. Keblinski [21], opined that the heat transferred by nanoparticle diffusion contributes minimally to thermal conductivity enhancement. In all, errors arising from preparation of nanofluids include, heating and measurement processes and cleanliness of apparatus are some of experimental results differences in the various studies while over simplification and assumption of certain parameters are the causes of some classical models. As a result of these uncertainties, some of the results appear to be conflicting. The aim of this study is to include Brownian motion into our proposed model which is a modification of the classical models of nanofluids of Koo and Kleinstreuer ([18 and 19]). By this modification, different sphericities for different shapes nanoparticles in addition to other consideration which is an extension are made.

Formalism

A steady boundary layer flow of viscous, incompressible, Newtonian Spherical nanofluid is considered. Using the Boussineq's approximation, the governing equations of the nanofluid in steady spherical coordinate is given as

$$\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{2v_r}{r'^2}+\frac{4}{r'}\frac{\partial v_r}{\partial r'}+\frac{\partial^2 v_r}{\partial r'^2}\right)+g\beta_{nf}\left(T-T_0\right)=0 \quad (1)$$

$$\frac{k_{nf}}{\left(\rho C_p\right)_{nf}}\left(\frac{2}{r'}\frac{\partial T}{\partial r'}+\frac{\partial^2 T}{\partial r'^2}\right)=0 \quad (2)$$

Subject to the boundary conditions (Wang et al, [9])

$$v_r\big|_{r'=R} = v_0$$
$$v_r\big|_{r'=0} = v_0 \quad (3)$$

$$T\big|_{r'=R} = T_w$$
$$T\big|_{r'=0} = T_0 \quad (4)$$

where v_r is nanofluid velocity, v_0 is characteristic velocity, r is radius of nanoparticles, T is temperature of nanofluid, T_0 is characteristic temperature, T_w is ambient temperature and g is acceleration due to gravity.

In this work, according to Hamilton and Crosser model [6], effective dynamic viscosity ratio and effective thermal conductivity ratio which are valid for both spherical and non spherical shapes nanoparticles are stated as

$$\frac{\mu_{nf}}{\mu_f} = \left(1 + a\phi + b\phi^2\right) \quad (5)$$

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} \quad (6)$$

The **n** is the empirical shape factor given by $n = \frac{3}{\psi}$ where ψ is the sphericity.

According to the work of Tiwari and Das [22] and Asma et al [23], density of nanofluid (ρ_{nf}), thermal expansion due to temperature of nanofluid (β_{nf}), specific heat at constant pressure of nanofluid are respectively

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$
$$\beta_{nf} = (1-\phi)\beta_f + \phi\beta_s \quad (7)$$
$$(C_p)_{nf} = (1-\phi)(C_p)_f + \phi(C_p)_s$$

where ϕ is the nanoparticles volume fractions given by $\phi = \frac{v_s}{v_f + v_s} = m\frac{\pi}{6}D_s^3$, m is the number of particles per unit volume and D is the average diameter of the particles, ρ_f and ρ_s are the densities of the base fluid and solid nanoparticles, β_f and β_s are the thermal expansion due to temperature of base fluid and solid nanoparticles, and $(C_p)_f$ and $(C_p)_s$ are the specific heat at constant pressure due to base fluid and solid nanoparticles. **a** and **b** are constants that depend on the particle shape (Aaiza et al [24]). The thermo physical properties of alumina (Al_2O_3) and copper (Cu) nanoparticles as well as water (H_2O) and ethylene glycol ($C_2H_6O_2$) as base fluids are presented in table 3.

Table 1: Sphericity ψ and empirical shape factor for different shapes nanoparticles (Aaiza et al [24])

Model	Platelet	Blade	Cylinder	Brick
ψ	0.52	0.36	0.62	0.81
n	5.76923	8.33333	4.83871	3.70370

Table 2: Constants **a** and **b** empirical shape factors (Aaiza et al [24])

Model	Platelet	Blade	Cylinder	Brick
a	37.1	14.6	13.5	1.9
b	612.6	123.3	904.4	471.4

Table 3: Thermo physical properties of Al_2O_3 and Cu nanoparticles, $C_2H_6O_2$ and H_2O (Aaiza et al [24])

Property	H_2O	$C_2H_6O_2$	Al_2O_3	Cu
$(C_p)_{nf}$ (J/kgK)	4179	0. 58	765	385
ρ (kg/m^3)	997.1	1.115	3970	8933
k_{nf} (W/mk)	0.613	0.149	40	401
$\mu_{nf}\ (m^2s^{-1})$	0.00089	0.001095	0.0000029014	0.00046
$\beta\ x10^{-5}(k^{-1})$	21	6. 5	0.85	1.67

The Hamilton and Crosser model [6], shows that the effective thermal conductivity of nanofluids depends on the thermal conductivity of the spherical solid, the base fluid and the volume fraction of the solid parts. However, to effectively determine the thermal conductivity of nanofluids, Patel et al [12], observe that, the Brownian motion effect on nanoparticles at the molecular and nano-scale levels may be a key mechanism governing the thermal behaviour of nanofluids. In a similar experimental study conducted by Das et al [13], they find out that thermal conductivity of nanofluids depends strongly on temperature and that this fact should be considered in theoretical models. The Hamilton and Crosser model ignored Brownian motion which is temperature dependent; it is therefore defective in determining the effective thermal conductivity of nanofluids. In a related development, studies that incorporate Brownian motion were reported. Few of the works include Xuan et al [14], which included the Brownian motion into the model proposed but was criticized of being too weak in temperature dependence and also at variance with experimental data of Das et al [13]. Also, Kumar et al [15], proposed a model to account for the large enhancement of thermal conductivity of nanofluids and its strong temperature dependence, but its use showed that it is not suitable for high concentration of particles. Another study carried out by Bhattacharya et al [16], developed a technique to determine the effective conductivity of a nanofluid using Brownian motion simulation. Although the model showed good agreement with other works, its sphericity is just 1. The same issue of sphericity being 1 was the weak point of the model proposed by Putnam et al [17] which is a modification of Maxwell's model. Above all, Koo and Kleinstrever ([18 and 19]), proposed a model of nanofluids which among other factors included the effect of particle size, particle volume fraction, temperature and properties of the base fluid. The model is stated as

$$\frac{k_{nf}}{k_f}=\frac{k_s+2k_f+2(k_s-k_f)\phi}{k_s+2k_f-(k_s-k_f)\phi}+5x10^4\xi(\rho C_p)_s\sqrt{\frac{k_BT}{\rho_sD}}f(T,\phi) \qquad (8)$$

where ξ and k_B are respectively, related to particle motion and Stefan- Boltzmann constant. The function $f(T,\phi)$ can vary continuously with particle volume fraction as $f(T,\phi)=(-6.04\phi+0.4705)T+(1722.30\phi-134.63)$. Comparing equation (6) with equation (8), it is realized that the first part of equation (8) is a special case with sphericity 1 of equation (6). To tackle the effect of heat transfer, sphericity and temperature dependent Brownian motion of Cu and Al_2O_3 nanofluids, equation (8) is modified to take the form

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} + 5x10^4 \xi\phi(\rho C_p)_s \sqrt{\frac{k_B T}{\rho_s D}} f(T,\phi) \quad (9)$$

2. DIMENSIONAL ANALYSIS

To effectively tackle the governing fluid flow equations, dimensional homogeneity of the governing equations using the Buckinham-$\pi -$theorem is stated

$$u = \frac{v_r t'}{D}, r = \frac{r'}{D}, , (\mathrm{Re}^{-1}) = \frac{\mu_f}{v_r \rho_f D}, \theta = \frac{T - T_0}{T_0 - T_w}, \mathrm{Pr} = \frac{v_r (\rho C_p)_f}{k_f}, G_\theta = \frac{g\beta_f (T - T_0)\mu_f}{v_r^3}$$

Also, substitute equation (9) into equation (2) and equation (5) into equation (1), the resulting equations transformed into

$$\frac{(1 + a\phi + b\phi^2)}{\mathrm{Re}}\left(\frac{2u}{r^2} + \frac{4}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \left((1-\phi) + \phi\frac{\beta_s}{\beta_f}\right) G_\theta \theta = 0 \quad (10)$$

$$\frac{\delta}{\mathrm{Pr}}\left(\frac{2}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial r^2}\right) = 0 \quad (11)$$

Subject to the boundary conditions

$$u|_{r=1} = 0, \quad u|_{r=0} = 0 \quad (12)$$

$$\theta|_{r=1} = 1, \quad \theta|_{r=0} = 0 \quad (13)$$

where **Re** is Reynolds' number, **Pr** is Prandtl number, G_θ is Grashofs number, θ is dimensionless temperature, **u** is dimensionless velocity and **r** is dimensionless radius of the nanoparticles.

$$\delta = \frac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi} + 5x10^4 \xi\phi(\rho C_p)_s \sqrt{\frac{k_B \theta}{\rho_s D}} f(T,\phi)$$

Simplification of equation (11) results in

$$\frac{\left(\alpha_1 + 5x10^4 \xi\phi(\rho C_p)_s \sqrt{\frac{k_B}{\rho_s D}}\left((-6.04\phi + 0.4705)\theta^{1.5} + (1722.3\phi - 134.63)\theta^{0.5}\right)\right)}{\mathrm{Pr}}\left(\frac{2}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial r^2}\right) = 0 \quad (14)$$

where $\alpha_1 \Pr = \dfrac{k_s + (n-1)k_f + (n-1)(k_s - k_f)\phi}{k_s + (n-1)k_f - (k_s - k_f)\phi}$

Method of Solution

Approximate $\theta^{1.5}$ and $\theta^{0.5}$ using Taylor's series expansion about 1 and neglect powers of θ ≥ 2 and simplify, equation (14) can be written as

$$\left(\alpha_1 + a_1(3\theta - 1) + a_2(\theta + 1)\right)\left(\frac{2}{r}\frac{\partial \theta}{\partial r} + \frac{\partial^2 \theta}{\partial r^2}\right) = 0 \qquad (15)$$

where

$$a_1 = \frac{\left(5x10^4 \xi\phi(\rho C_p)_s \sqrt{\dfrac{k_B}{\rho_s D}}(-6.04\phi + 0.4705)\right)}{2\Pr}$$

$$a_2 = \frac{\left(5x10^4 \xi\phi(\rho C_p)_s \sqrt{\dfrac{k_B}{\rho_s D}}(1722.3\phi - 134.63)\right)}{2\Pr}$$

The simplification of equation (15) and its solution as well as imposing the boundary conditions of equation (13), results in

$$\theta^2(3a_1 + a_2) + \theta(\alpha_1 - a_1 + a_2) - (\alpha_1 + 2a_1 + 2a_2)r = 0 \qquad (16)$$

and finally reduced to

$$\theta(r) = \frac{-(\alpha_1 - a_1 + a_2) \pm \sqrt{(\alpha_1 - a_1 + a_2)^2 + 4(3a_1 + a_2)((\alpha_1 + 2a_1 + 2a_2)r)}}{2(3a_1 + a_2)} \qquad (17)$$

Since $\theta > 0$, $+\theta$ is chosen.

To solve equation (10), equation (17) is put into it and the resulting expression is simplified as

$$\left(\frac{2u}{r^2} + \frac{4}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2}\right) + \beta_1\left(\frac{\beta_2 + \sqrt{\beta_2^2 + \beta_3 r}}{\beta_4}\right) = 0 \qquad (18)$$

The resulting solution of equation (18), after imposing the boundary conditions of equation (13) is given as

$$u(r) = \frac{\beta_1 r^2}{2\beta_4} - \frac{4\left(\sqrt{\beta_2^2 + \beta_3 r}\right)^5}{15\beta_4\beta_3^2} + \left(\frac{4\left(\sqrt{\beta_2^2 + \beta_3}\right)^5}{15\beta_4\beta_3^2} - \frac{1}{2}\right)r + \frac{4\left(\sqrt{\beta_2^2}\right)^5}{15\beta_4\beta_3^2} \qquad (19)$$

where

$$\beta_1 = \frac{\left((1-\phi)+\phi\frac{\beta_s}{\beta_f}\right)\mathrm{Re}\,G_\theta}{\left(1+a\phi+b\phi^2\right)},\quad \beta_2 = (\alpha_1 - a_1 + a_2),\ \beta_3 = 4(3a_1 + a_2)(\alpha_1 + 2a_1 + 2a_2),\ \beta_4 = 2(3a_1 + a_2)$$

The mass flux ϖ and the mean temperature θ_m are obtained by evaluating the integral

$$\varpi = \int_0^1 u(r)dr \qquad (20)$$

and

$$\theta_m = \frac{\int_0^1 u(r)\theta(r)dr}{\int_0^1 u(r)dr} \qquad (21)$$

Heat Transfer Coefficient (Nu)

It has been shown that the heat transfer performance is a better indicator than the effective thermal conductivity of nanofluids, particularly when such nanofluids are used as coolants and other functions in industries. This property necessitated the formation of nanofluid models to either complement the models of thermal conductivity or an improved determination of effective thermal conductivity. Following the models for determining the heat transfer coefficients of nanofluids (Polidori at al [25], Mansour et al [26]), the mean Nusselt number is calculated thus,

$$\left.\frac{\partial\theta(r)}{\partial r}\right|_{r=0} = Nu = \frac{\beta_3}{2\beta_4\sqrt{\beta_2^2}} \qquad (20)$$

Skin Friction

The skin friction is given by

$$\left.\frac{\partial u(r)}{\partial r}\right|_{r=0} = \tau = \frac{2\beta_3}{3\beta_4\beta_3^2\left(\sqrt{\beta_2^2}\right)^3} + \left(\frac{4\left(\sqrt{\beta_2^2+\beta_3}\right)^5}{15\beta_4\beta_3^2} - \frac{1}{2}\right) \qquad (21)$$

3. RESULTS AND DISCUSSION

Results

Table 4: Numerical values of mean Nusselt number for various values of Prandtl number in H_2O and $C_2H_6O_2$ based fluids when

$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \phi = 0.06mm^3$

Pr	$Nu(Al_2O_3)C_2H_6O_2$	$Nu(Al_2O_3)H_2O$	**Nu**$(Cu)C_2H_6O_2$	**Nu**$(Cu)H_2O$
0.71	$-3.60599\,x10^{-12}$	$-3.48063\,x10^{-12}$	$2.07758\,x10^{-12}$	$2.06972\,x10^{-12}$

1.71	$-3.60597\, x10^{-12}$	$-3.48062\, x10^{-12}$	$2.07757\, x10^{-12}$	$2.06971\, x10^{-12}$
2.71	$-3.60593\, x10^{-12}$	$-3.48058\, x10^{-12}$	$2.07756\, x10^{-12}$	$2.06969\, x10^{-12}$
3.71	$-3.60588\, x10^{-12}$	$-3.48053\, x10^{-12}$	$2.07753\, x10^{-12}$	$2.06967\, x10^{-12}$

Table 5: Numerical values of mean Nusselt number for various values of nanoparticles volume fractions in H_2O and $C_2H_6O_2$ based fluids when

$k_B = 1.380658\, x10^{-23}\, JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71$

$\phi(mm^3)$	Nu(Al_2O_3) $C_2H_6O_2$	Nu(Al_2O_3) H_2O	**Nu(** Cu) $C_2H_6O_2$	**Nu(** Cu) H_2O
0.06	$3.60599\, x10^{-12}$	$3.48063\, x10^{-12}$	$2.07758\, x10^{-12}$	$2.06972\, x10^{-12}$
0.07	$9.80488\, x10^{-13}$	$9.41793\, x10^{-13}$	$5.6575\, x10^{-13}$	$5.63316\, x10^{-12}$
0.08	$1.65227\, x10^{-13}$	$1.57965\, x10^{-13}$	$9.54736\, x10^{-14}$	$9.50156\, x10^{-14}$
0.09	$5.51207\, x10^{-12}$	$5.24614\, x10^{-12}$	$3.18945\, x10^{-12}$	$3.17262\, x10^{-12}$

Table 6: Numerical values of mean Nusselt number for various values of nanoparticles diameter in H_2O and $C_2H_6O_2$ based fluids when

$k_B = 1.380658\, x10^{-23}\, JK^{-1}, \xi = 1, \theta = 25, \phi = 0.06mm^3, \Pr = 0.71$

d(nm)	Nu(Al_2O_3) $C_2H_6O_2$	Nu(Al_2O_3) H_2O	**Nu(** Cu) $C_2H_6O_2$	**Nu(** Cu) H_2O
15	$3.60599\, x10^{-12}$	$3.48063\, x10^{-12}$	$2.07758\, x10^{-12}$	$2.06972\, x10^{-12}$
17	$4.08679\, x10^{-12}$	$3.94472\, x10^{-12}$	$2.35459\, x10^{-12}$	$2.34568\, x10^{-12}$
19	$4.56759\, x10^{-12}$	$4.4088\, x10^{-12}$	$2.6316\, x10^{-12}$	$2.62164\, x10^{-12}$
21	$5.04838\, x10^{-12}$	$4.87289\, x10^{-12}$	$2.90861\, x10^{-12}$	$2.8976\, x10^{-12}$

Table 7: Numerical values of mean Nusselt number for various values of nanoparticles Sphericity in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\, x10^{-23}\, JK^{-1}, \xi = 1, \theta = 25, \phi = 0.06mm^3, \Pr = 0.71, d(nm) = 15$

ψ	Nu(Al_2O_3) $C_2H_6O_2$	Nu(Al_2O_3) H_2O	**Nu(** Cu) $C_2H_6O_2$	**Nu(** Cu) H_2O
0.36	$4.47397\, x10^{-12}$	$4.19821\, x10^{-12}$	$2.60158\, x10^{-12}$	$2.58334\, x10^{-12}$
0.52	$3.60599\, x10^{-12}$	$3.48063\, x10^{-12}$	$2.07758\, x10^{-12}$	$2.06972\, x10^{-12}$
0.62	$3.31023\, x10^{-12}$	$3.22403\, x10^{-12}$	$1.90174\, x10^{-12}$	$1.89643\, x10^{-12}$
0.81	$2.96389\, x10^{-12}$	$2.91493\, x10^{-12}$	$1.69762\, x10^{-12}$	$1.69467\, x10^{-12}$

Table 8: Numerical values of Skin friction for various values of Prandtl number in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \phi = 0.06mm^3$

Pr	$\tau\ (Al_2O_3)C_2H_6O_2$	$\tau\ (Al_2O_3)H_2O$	$\tau\ (Cu)C_2H_6O_2$	$\tau\ (Cu)H_2O$
0.71	$-1.70014\,x10^{17}$	$-1.61226\,x10^{17}$	$-4.01659\,x10^{17}$	$-3.99391\,x10^{17}$
1.71	$-8.7099\,x10^{14}$	$-8.25961\,x10^{14}$	$-2.05783\,x10^{15}$	$-2.04616\,x10^{15}$
2.71	$-5.47952\,x10^{13}$	$-5.194\,x10^{13}$	$-1.29578\,x10^{14}$	$-1.28839\,x10^{14}$
3.71	$-8.00715\,x10^{12}$	$-7.55025\,x10^{12}$	$-1.91401\,x10^{13}$	$-1.90237\,x10^{13}$

Table 9: Numerical values of skin friction for various values of nanoparticles volume fractions in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71$

$\phi(mm^3)$	$\tau\ (Al_2O_3)C_2H_6O_2$	$\tau\ (Al_2O_3)H_2O$	$\tau\ (Cu)C_2H_6O_2$	$\tau\ (Cu)H_2O$
0.06	$-1.70014\,x10^{17}$	$-1.61226\,x10^{17}$	$-4.01659\,x10^{17}$	$-3.99391\,x10^{17}$
0.07	$-1.58285\,x10^{18}$	$-1.49008\,x10^{18}$	$-3.74798\,x10^{18}$	$-3.72381\,x10^{18}$
0.08	$3.00003\,x10^{19}$	$2.80444\,x10^{19}$	$7.11892\,x10^{19}$	$7.06775\,x10^{19}$
0.09	$2.0286\,x10^{17}$	$1.88359\,x10^{17}$	$4.82372\,x10^{17}$	$4.78561\,x10^{18}$

Table 10: Numerical values of skin friction for various values of nanoparticles diameter in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, \phi = 0.06mm^3, \Pr = 0.71$

d(nm)	$\tau\ (Al_2O_3)C_2H_6O_2$	$\tau\ (Al_2O_3)H_2O$	$\tau\ (Cu)C_2H_6O_2$	$\tau\ (Cu)H_2O$
15	$-1.70014\,x10^{17}$	$-1.61226\,x10^{17}$	$-4.01659\,x10^{17}$	$-3.99391\,x10^{17}$
17	$-1.40912\,x10^{17}$	$-1.33629\,x10^{17}$	$-3.32914\,x10^{17}$	$-3.31026\,x10^{17}$
19	$-1.19255\,x10^{17}$	$-1.13095\,x10^{17}$	$-2.81757\,x10^{17}$	$-2.80159\,x10^{17}$
21	$-1.02634\,x10^{17}$	$-9.73293\,x10^{16}$	$-2.4248\,x10^{17}$	$-2.41105\,x10^{17}$

Table 11: Numerical values of skin friction for various values of nanoparticles Sphericity in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, \phi = 0.06mm^3, \Pr = 0.71, d(nm) = 15$

ψ	$\tau\ (Al_2O_3)C_2H_6O_2$	$\tau\ (Al_2O_3)H_2O$	$\tau\ (Cu)C_2H_6O_2$	$\tau\ (Cu)H_2O$

0.36	$-2.34957\,x10^{17}$	$-2.13572\,x10^{17}$	$-5.62845\,x10^{17}$	$-5.56934\,x10^{17}$
0.52	$-1.70014\,x10^{17}$	$-1.61226\,x10^{17}$	$-4.01669\,x10^{17}$	$-3.99391\,x10^{17}$
0.62	$-1.49532\,x10^{17}$	$-1.4373\,x10^{17}$	$-3.51769\,x10^{17}$	$-3.50299\,x10^{17}$
0.81	$-1.2669\,x10^{17}$	$-1.23563\,x10^{17}$	$-2.96682\,x10^{17}$	$-2.95911\,x10^{17}$

Table 12: Numerical values of Thermal conductivity ratio for various values of nanoparticles volume fractions in H_2O and $C_2H_6O_2$ based fluids when

$$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71$$

$\phi(mm^3)$	$\frac{k_{nf}}{k_f}$ (Al_2O_3) $C_2H_6O_2$	$\frac{k_{nf}}{k_f}$ $(Cu)\,C_2H_6O_2$	$\frac{k_{nf}}{k_f}$ (Cu) H_2O	$\frac{k_{nf}}{k_f}$ $(Al_2O_3)\,H_2O$
0.06	-29.9320	-29.9246	-30.2434	-30.2472
0.07	-12.6446	-12.6358	-13.0116	-13.0162
0.08	4.64418	4.65451	4.22025	4.21494
0.09	21.9344	21.9462	21.4523	21.4462

Table 13: Computed values of Thermal conductivity ratio for various values of nanoparticles Temperature in H_2O and $C_2H_6O_2$ based fluids when

$$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \phi = 0.06mm^3, d = 15nm, \Pr = 0.71$$

θ	$\frac{k_{nf}}{k_f}$ (Cu) $C_2H_6O_2$	$\frac{k_{nf}}{k_f}$ $(Al_2O_3)\,C_2H_6O_2$	$\frac{k_{nf}}{k_f}$ (Cu) H_2O	$\frac{k_{nf}}{k_f}$ $(Al_2O_3)\,H_2O$
25	-29.9246	-29.932	-30.2434	-30.2472
50	-29.9246	-29.932	-30.2434	-30.2472
100	-29.9246	-29.932	-30.2434	-30.2472
1000	-29.9244	-29.9318	-30.2432	-30.2470

Table 14: Numerical values of Thermal Conductivity ratio for various values of nanoparticles diameter in H_2O and $C_2H_6O_2$ based fluids when

$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, \phi = 0.06mm^3, \Pr = 0.71$

$d(nm)$	$\frac{k_{nf}}{k_f}(Al_2O_3)C_2H_6O_2$	$\frac{k_{nf}}{k_f}(Cu)C_2H_6O_2$	$\frac{k_{nf}}{k_f}(Cu)H_2O$	$\frac{k_{nf}}{k_f}(Al_2O_3)H_2O$
15	-29.932	-29.9246	-30.2434	-30.2472
17	-29.932	-29.9246	-30.2434	-30.2472
19	-29.932	-29.9246	-30.2434	-30.2472
21	-29.932	-29.9246	-30.2434	-30.2472

Table 15: Numerical values of Thermal conductivity ratio for various values of sphericity in $C_2H_6O_2$ and H_2O based fluids when

$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \phi = 0{,}06mm^3\ \Pr = 0.71$

ψ	$\frac{k_{nf}}{k_f}(Al_2O_3)C_2H_6O_2$	$\frac{k_{nf}}{k_f}(Cu)C_2H_6O_2$	$\frac{k_{nf}}{k_f}(Al_2O_3)H_2O$	$\frac{k_{nf}}{k_f}(Cu)H_2O$
0.36	-29.7771	-29.7618	-29.8246	-29.7672
0.52	-29.9320	-29.9246	-29.9559	-29.9272
0.62	-29.9890	-29.9837	-30.0061	-29.9856
0.81	-30.0590	-30.0559	-30.0693	-30.057

Table 16: Numerical values of Temperature profile for various values of nanoparticles volume fractions in H_2O and $C_2H_6O_2$ based fluids when

$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71$

$\phi(mm^3)$	$\theta(r)(Al_2O_3)C_2H_6O_2$	$\theta(r)\ (Al_2O_3)H_2O$	$\theta(r)\ (Cu)C_2H_6O_2$	$\theta(r)\ (Cu)H_2O$
0.06	$3.65787\,x10^{-21}$	$3.65787\,x10^{-21}$	$2.11613\,x10^{-21}$	$2.11513\,x10^{-21}$
0.07	$9.29939\,x10^{-22}$	$9.29939\,x10^{-22}$	$5.26524\,x10^{-22}$	$5.26524\,x10^{-22}$
0.08	$1.52039\,x10^{-22}$	$1.52039\,x10^{-22}$	$6.88666\,x10^{-23}$	$9.18222\,x10^{-23}$
0.09	$4.20135\,x10^{-21}$	$4.3694\,x10^{-21}$	$2.41049\,x10^{-21}$	$2.53735\,x10^{-21}$

Table 17: Numerical values of Temperature profile for various values of Prandtl number in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \phi = 0.06mm^3$

Pr	$\theta(r)$ $(Al_2O_3)C_2H_6O_2$	$\theta(r)(Al_2O_3)H_2O$	$\theta(r)(Cu)C_2H_6O_2$	$\theta(r)(Cu)H_2O$
0.71	$3.65787\,x10^{-21}$	$3.65987\,x10^{-21}$	$2.11514\,x10^{-21}$	$2.11513\,x10^{-21}$
1.71	$2.13685\,x10^{-20}$	$2.9941\,x10^{-20}$	$1.21694\,x10^{-20}$	$1.22402\,x10^{-20}$
2.71	$5.37676\,x10^{-20}$	$7.52176\,x10^{-20}$	$3.0611\,x10^{-20}$	$3.0611\,x10^{-20}$
3.71	$1.0055\,x10^{-19}$	$1.40866\,x10^{-19}$	$5.74104\,x10^{-20}$	$5.74104\,x10^{-20}$

Table 18: Numerical values of Temperature profile for various values of nanoparticles diameter in H_2O and $C_2H_6O_2$ based fluids when
$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71$

d(nm)	$\theta(r)$ $(Al_2O_3)C_2H_6O_2$	$\theta(r)$ $(Al_2O_3)H_2O$	$\theta(r)$ $(Cu)C_2H_6O_2$	$\theta(r)$ $(Cu)H_2O$
15	$3.65787\,x10^{-21}$	$3.65787\,x10^{-21}$	$2.11513\,x10^{-21}$	$2.11513\,x10^{-21}$
17	$4.22552\,x10^{-21}$	$4.14266\,x10^{-21}$	$2.37682\,x10^{-21}$	$2.37682\,x10^{-21}$
19	$4.64235\,x10^{-21}$	$4.64235\,x10^{-21}$	$2.64499\,x10^{-21}$	$2.64499\,x10^{-21}$
21	$5.15683\,x10^{-21}$	$5.15683\,x10^{-21}$	$2.91976\,x10^{-21}$	$2.64499\,x10^{-21}$

Table 19: Numerical values of Temperature profile for various values of nanoparticles Sphericity in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, \phi = 0.06mm^3, \Pr = 0.71, d(nm) = 15$

ψ	$\theta(r)$ $(Al_2O_3)C_2H_6O_2$	$(\theta(r)$ $Al_2O_3)H_2O$	$\theta(r)$ $(Cu)C_2H_6O_2$	$\theta(r)$ $(Cu)H_2O$
0.36	$3.7357\,x10^{-21}$	$3.7351\,x10^{-21}$	$2.11513\,x10^{-21}$	$2.11513\,x10^{-21}$
0.52	$3.65787\,x10^{-21}$	$3.65787\,x10^{-21}$	$2.11513\,x10^{-21}$	$2.11513\,x10^{-21}$
0.62	$3.65787\,x10^{-21}$	$3.65787\,x10^{-21}$	$2.11513\,x10^{-21}$	$2.11513\,x10^{-21}$
0.81	$3.7357\,x10^{-21}$	$3.65787\,x10^{-21}$	$2.11513\,x10^{-21}$	$2.11513\,x10^{-21}$

Table 20: Numerical values of Velocity profile for various values of nanoparticles volume fractions in H_2O and $C_2H_6O_2$ based fluids when

$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71, \mathrm{Re} = 1000, G_\theta = 0.92$

$\phi(mm^3)$	$u(r)\,(Al_2O_3)\,C_2H_6O_2$	$u(r)\,(Al_2O_3)\,H_2O$	$u(r)\,(Cu)\,C_2H_6O_2$	$u(r)\,(Cu)\,H_2O$
0.06	$-1.27511\,x10^9$	$-1.2092\,x10^9$	$-3.01253\,x10^9$	$-2.99544\,x10^9$
0.07	$-1.18714\,x10^{10}$	$-1.11756\,x10^{10}$	$-2.81099\,x10^{10}$	$-2.79286\,x10^{10}$
0.08	$2.25002\,x10^{11}$	$2.10333\,x10^{11}$	$5.33919\,x10^{11}$	$5.30081\,x10^{11}$
0.09	$1.52145\,x10^9$	$1.41268\,x10^9$	$3.61778\,x10^9$	$3.58919\,x10^9$

Table 21: Numerical values of Velocity profile for various values of Prandtl number in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \mathrm{Re} = 1000, G_\theta = 0.92$

Pr	$u(r)$ $(Al_2O_3)\,C_2H_6O_2$	$u(r)\,(Al_2O_3)\,H_2O$	$u(r)\,(Cu)\,C_2H_6O_2$	$u(r)\,(Cu)\,H_2O$
0.71	$-1.27511\,x10^9$	$-1.2092\,x10^9$	$-3.01253\,x10^9$	$-2.99544\,x10^9$
1.71	$-6.53311\,x10^6$	$-6.19543\,x10^6$	$-1.54349\,x10^7$	$-1.53474\,x10^7$
2.71	-412363	-391048	-974235	-968710
3.71	-62639.7	-59401.9	-147991	-147151

Table 22: Numerical values of Velocity profile for various values of nanoparticles diameter in H_2O and $C_2H_6O_2$ based fluids when
$k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, \Pr = 0.71, \mathrm{Re} = 1000, G_\theta = 0.92$

d(nm)	$u(r)\,(Al_2O_3)\,C_2H_6O_2$	$u(r)\,(Al_2O_3)\,H_2O$	$u(r)\,(Cu)\,C_2H_6O_2$	$u(r)\,(Cu)\,H_2O$
15	$-1.27511\,x10^9$	$-1.2092\,x10^9$	$-3.01253\,x10^9$	$-2.99544\,x10^9$
17	$-1.05684\,x10^9$	$-1.00222\,x10^9$	$-2.4968\,x10^9$	$-2.4827\,x10^9$
19	$-8.94446\,x10^8$	$-8.48214\,x10^8$	$-2.11319\,x10^9$	$-2.1012\,x10^9$
21	$-7.6976\,x10^8$	$-7.29973\,x10^8$	$-1.81861\,x10^9$	$-1.80829\,x10^9$

Table 23: Numerical values of Velocity profile for various values of nanoparticles Sphericity in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71, \mathrm{Re} = 1000, G_\theta = 0.92$

ψ	$u(r)$	$u(r)\,(Al_2O_3)\,H_2O$	$u(r)\,(Cu)\,C_2H_6O_2$	$u(r)\,(Cu)\,H_2O$

	$(Al_2O_3)C_2H_6O_2$			
0.36	$-1.76218\,x10^{9}$	$-1.6018\,x10^{9}$	$-4.22135\,x10^{9}$	$-4.17702\,x10^{9}$
0.52	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
0.62	$-1.12151\,x10^{9}$	$-1.07798\,x10^{9}$	$-2.63828\,x10^{9}$	$-2.62725\,x10^{9}$
0.81	$-9.50177\,x10^{8}$	$-9.26728\,x10^{8}$	$-2.22512\,x10^{9}$	$-2.21934\,x10^{9}$

Table 24: Numerical values of Velocity profile for various values of Reynolds' number in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71, G_\theta = 0.92$

Re	$u(r)$ $(Al_2O_3)C_2H_6O_2$	$u(r)$ $(Al_2O_3)H_2O$	$u(r)$ $(Cu)C_2H_6O_2$	$u(r)$ $(Cu)H_2O$
1000	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
2000	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
3000	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
4000	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$

Table 25: Numerical values of Velocity profile for various values of Grashofs number in H_2O and $C_2H_6O_2$ based fluids when $k_B = 1.380658\,x10^{-23}\,JK^{-1}, \xi = 1, \theta = 25, d = 15nm, \Pr = 0.71, \mathrm{Re} = 1000,$

G_θ	$u(r)$ $(Al_2O_3)C_2H_6O_2$	$u(r)$ $(Al_2O_3)H_2O$	$u(r)$ $(Cu)C_2H_6O_2$	$u(r)$ $(Cu)H_2O$
0.92	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
1.92	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
2.92	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$
4000	$-1.27511\,x10^{9}$	$-1.2092\,x10^{9}$	$-3.01253\,x10^{9}$	$-2.99544\,x10^{9}$

Table 26: Computed values of Viscosity ratio of different shapes nanoparticles for various values of nanoparticles volume fractions

$\phi(mm^3)$	*Platelet*	*Blade*	*Cylinder*	*Brick*
0.06	5.43136	2.31988	5.06584	2.81104

0.07	6.59874	2.62617	6.37656	3.44286
0.08	7.88864	2.95712	7.86816	4.16896
0.09	9.30106	3.31273	9.54064	4.98934

4. DISCUSSION

Mean Nusselt number

Table 4 displayed the effect of Prandtl number on the heat transfer coefficient of Cu and Al_2O_3 nanoparticles in H_2O and $C_2H_6O_2$ based fluids. It shows that increase in Pr, corresponds to an increase in the nanofluids of Cu and Al_2O_3 in both H_2O and $C_2H_6O_2$. However, the magnitude of the nanofluid of Cu is greater than nanofluid of Al_2O_3. Table 5, numerically showed that increase in nanoparticles volume fractions, enhanced the heat transfer coefficient of the nanoparticles in the two base fluids under consideration and this observation is consistent with the work of Polidori et al [26] and Abu-Nada [27]. The nanoparticles diameter is an important parameter in the enhancement of heat transfer coefficient and this fact is clearly demonstrated in Table 6. Almost all literature cited, established this fact. Increase in nanoparticles sphericity as depicted in Table 7, showed a decrease in the heat transfer coefficient. This is because mixture of nanoparticles with base fluids delayed diffusion and this in turn affect the Nusselt number.

Skin friction

Table 8, displayed that, as the Prandtl number increases, the skin friction within the nanofluids of Cu and Al_2O_3 in H_2O and $C_2H_6O_2$ base fluids increases. However, Al_2O_3 nanofluid increase in magnitude appreciates better than Cu nanofluid.

Table 9, showed that the skin friction increases as the nanoparticle volume fraction increases but beyond $0.08mm^3$ fraction, a decrease was observed. The explanation is that to improve the skin friction of nanofluids particularly Cu and Al_2O_3 nanofluids, care must be taken to ensure the choice of the nanoparticle volume fractions. It is a truism that increase in skin friction between molecules nanofluids is enhanced by the size of the nanoparticles and Table 10 described it vividly in the two nanofluids under consideration.

The skin friction increases as a result of increase in the sphericity of nanoparticles in the nanofluids. However, the increase is higher in water based fluids as shown in Table 11. This observation is consistent with the work of Temovera et al [28].

Thermal conductivity ratio

Table 12 displayed the effect of increasing nanoparticle volume fractions on thermal conductivity of Cu and Al_2O_3 nanofluids. It shows that increase in volume fractions led to an increase in both the water and ethylene glycol based fluids. This result is in agreement with that of Lee and Choi [29]. Table 13, showed that an increase in temperature up to about 373K does not show any appreciable enhancement in thermal conductivity of Cu and Al_2O_3 nanofluids. This assertion was also corroborated by Li and Williams [30], Kolade et al [31] and Williams et al [32]. However, at temperature of 1273K and above, an increase in thermal conductivity was observed.

An increase in nanoparticles diameter of Alumina and Copper in ethylene glycol and water based fluids as shown in Table 14, no increase in thermal conductivity was observed. From Table 15, an increase in sphericity results in a decrease in thermal conductivity enhancement but the decrease is more pronounced in $C_2H_6O_2$ based fluid than H_2O based fluid, Lee and Choi [29] and Wang et al [20] also reported same findings.

Temperature Profiles

Table 16, showed numerical values which indicated that increasing the volume fractions of Cu and Al_2O_3 nanofluids showed a decrease initially and later an increase in the temperature profile of the nanofluids but increasing the Prandtl number showed a corresponding increase in the temperature profile of the nanofluids as shown in Table 17. Table 18 showed that increasing the diameter of nanoparticles also increases the temperature profile of the nanofluids. A decrease was observed in temperature profile of the nanofluids as the sphericity is increased as shown in Table 19. in the absence of the sphericity, the observations are consistent with the works of Koo and Kleinstrever ([18 and 19]), though weak owing to the presence of Brownian motion

Velocity Profiles

Tables 20 – 23, respectively displayed the numerical values of increasing volume fractions, Prandtl number, nanoparticles diameter and sphericity of the two nanofluids, showed that the velocity profiles of the nanoofluids all increased considerably but surprisingly an increase in Reynolds' number(within the laminar and turbulence flow regime) and Grashof number due to temperature(which implies cooling the surface of the nanofluids) do not affect the velocity profiles of the nanofluids as shown in Table 24 and Table 25 respectively.

Table 26 clearly showed from the model that viscosity is largely dependent on nanoparticle volume fractions and therefore, its increase will no doubt corresponds to an increase in the viscosity of the nanofluids and it is supported by all the models listed in Wang and Mujumdar [33] and the work of Hashin and Shtrikman [34].

5. CONCLUSION

The study theoretically tackled thermal conductivity enhancement and heat transfer coefficient of two nanofluids, namely alumina and copper in water and ethylene glycol based fluids. Although, the numerical values used showed considerable agreement with some existing works cited (Xuan and Li [35], Sato et al [36], a departure was also observed (Wong and Leon [37], Kuznetsov and Nield [38]) in others maybe as a result of approach or approximations. One vital observation is that a model however novel may not incorporate all the essential ingredients or factors of a given nanofluid. Temperature is also a weak parameter in influencing the enhancement of thermal conductivity.

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Conflict of Interest

The authors declare that there are no conflicts of interests.

Peer-review

External peer-review was done through double-blind method.

Data and materials availability

All data associated with this study are present in the paper.

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