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Almost slightly semi-continuity, slightly semi-open and slightly semi-closed mappings

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ABSTRACT

In this paper we discuss new type of continuous functions called almost slightly semi−continuous, slightly semi-open and slightly semi-closed functions; its properties and interrelation with other such functions are studied.

Keywords: slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β−continuous functions and slightly v−continuous functions

AMS-classification Numbers: 54C10; 54C08; 54C05

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1. INTRODUCTION

In 1995 T. M. Nour introduced slightly semi-continuous functions. After him T. Noiri and G. I. Chae further studied slightly semi-continuous functions in 2000. T. Noiri individually studied about slightly β−continuous functions in 2001. C. W. Baker introduced slightly precontinuous functions in 2002. Arse Nagli Uresin and others studied slightly δ−continuous functions in 2007. Recently S. Balasubramanian and P.A.S. Vyjayanthi studied slightly v−continuous functions in 2011. Inspired with these developments we introduce in this paper almost slightly semi−continuous, slightly semi-open and slightly semi-closed functions and study its basic properties and interrelation with other type of such functions. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. PRELIMINARIES

Definition 2.1: $A\subset X$ is called g-closed[rg-closed] if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X.

Definition 2.2: A function f:X→ Y is said to be
(i) continuous[resp: nearly-continuous; rα-continuous; α-continuous; semi-continuous; β−continuous; pre-continuous] if inverse image of each open set is open[resp: regular-open; rα-open; α-open; semi-open; β -open; preopen].
(ii) almost continuous[resp: almost nearly-continuous; almost rα-continuous; almost α-continuous; almost semi-continuous; almost β−continuous; almost pre-continuous] if for each x in X and each open set (V, f(x)), ∃ an open[resp: regular-open; rα-open; α-open; semi-open; β-open; preopen] set (U, x) such that $f(U) \subset (cl(V))^o$.
(iii) slightly continuous[resp: slightly semi-continuous; slightly pre-continuous; slightly β−continuous; slightly α-continuous; slightly r-continuous; slightly v-continuous] at x in X if for each clopen subset V in Y containing f(x), $\exists U\in\tau(X)$[$\exists U\in SO(X)$; $\exists U\in PO(X)$; $\exists U\in\beta O(X)$; $\exists U\in\alpha O(X)$; $\exists U\in RO(X)$; $\exists U\in vO(X)$] containing x such that $f(U)\subseteq V$.

Lemma 2.1:

(i) Let A and B be subsets of a space X, if $A\in\tau(X)$ and $B\in RO(X)$, then $A\cap B\in\tau(B)$.
(ii)Let $A\subset B\subset X$, if $A\in\tau(B)$ and $B\in RO(X)$, then $A\in\tau(X)$.

Note 1: RCO(Y, f(x)) means regular-clopen set in Y containing f(x) and τ(X, x) means open set in X containing x.

3. ALMOST SLIGHTLY SEMI-CONTINUOUS FUNCTIONS

Definition 3.1: A function f:X→Y is said to be almost slightly semi-continuous at x in X if for each V∈RCO(Y, f(x)), ∃ U∈SO(X, x) such that f(U) ⊆ V and almost slightly semi-continuous if it is almost slightly semi-continuous at each x in X.

Note 2: Here after we call almost slightly semi-continuous function as al.sl.s.c function shortly.

Example 3.1: X = Y = {a, b, c}; τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ, {a}, {b, c}, Y}. Let f be defined as f(a) = b; f(b) = c and f(c) = a, then f is al.sl.s.c.

Example 3.2: X = Y = {a, b, c}; τ = {ϕ, {a}, {b, c}, X} and σ = {ϕ, {a}, {b}, {a, b}, Y}. Let f be defined as f(a) = b; f(b) = c and f(c) = a, then f is not al.sl.s.c.

Theorem 3.1: The following are equivalent:
(i) f is al.sl.s.c.
(ii) f^{-1}(V) is semi-open for every r-clopen set V in Y.
(iii) f^{-1}(V) is semi-closed for every r-clopen set V in Y.
(iv) f(scl(A)) ⊆ scl(f(A)).

Corollary 3.1: The following are equivalent.
(i) f is al.sl.s.c.
(ii) For each x in X and each V∈ RCO(Y, f(x)) ∃ U∈SO(X, x) such that f(U) ⊆ V.

Theorem 3.2: Let ∑ = {U_i:i∈ I} be any cover of X by regular open sets in X. A function f is al.sl.s.c. iff $f_{/U_i}$: is al.sl.s.c., for each i∈ I.
Proof: Let i∈ I be an arbitrarily fixed index and U_i∈ RO(X). Let x∈ U_i and V∈ RCO(Y, f_{U_i}(x)) Since f is al.sl.s.c, ∃ U∈SO(X, x) such that f(U) ⊂ V. Since U_i∈RO(X), by Lemma 2.1 x∈U∩U_i∈SO(U_i) and ($f_{/U_i}$)U∩U_i = f(U∩ U_i)⊂ f(U) ⊂ V. Hence $f_{/U_i}$ is al.sl.s.c. Conversely Let x in X and V∈RCO(Y, f(x)), ∃ i∈ I such that x∈U_i. Since $f_{/U_i}$ is al.sl.s.c, ∃ U∈SO(U_i, x) such that $f_{/U_i}$(U) ⊂ V. By Lemma 2.1, U∈SO(X) and f(U) ⊂ V. Hence f is al.sl.s.c.

Theorem 3.3: If f is almost continuous and g is continuous[al.sl.s.c.,], then g• f is al.sl.s.c.

Theorem 3.4: If f is almost continuous, open and g be any function, then g•f is al.sl.s.c iff g is al.sl.s.c.
Proof: If part: Theorem 3.3
Only if part: Let A∈RCO(Z). Then (g• f)$^{-1}$(A)∈τ(X). Since f is open, f(g• f)$^{-1}$(A) = g^{-1}(A) is open in Y. Thus g is al.sl.s.c.

Corollary 3.2: If f is r-irresolute, open and bijective, g is a function. Then g is al.sl.s.c. iff g• f is al.sl.s.c.

Theorem 3.5: If g: X → X× Y, defined by g(x) = (x, f(x)) for all x in X be the graph function of f:X→Y. Then g is al.sl.s.c iff f is al.sl.s.c.
Proof: Let V∈RCO(Y), then X×V∈RCO(X×Y). Since g is al.sl.s.c., f^{-1}(V) = f^{-1}(X×V)∈SO(X). Thus f is al.sl.s.c.
Conversely, let x in X and F∈RCO(X×Y, g(x)). Then F∩({x}×Y)∈RCO({x}×Y, g(x)). Also {x}×Y is homeomorphic to Y. Hence {y∈Y:(x, y)∈F}∈RCO(Y). Since f is al.sl.s.c. ∪{f^{-1}(y):(x, y)∈F} is open in X. Further x∈∪{f^{-1}(y):(x, y)∈F}⊆g^{-1}(F). Hence g^{-1}(F) is open. Thus g is al.sl.s.c.

Theorem 3.6: (i) f: $\Pi X_\lambda \to \Pi Y_\lambda$ is al.sl.s.c, iff f_λ: $X_\lambda \to Y_\lambda$ is al.sl.s.c for each λ∈Γ.
(ii) If f: X→ ΠY_λ is al.sl.s.c, then P_λ•f :X→ Y_λ is al.sl.s.c for each λ∈Γ, where P_λ:ΠY_λ onto Y_λ.

Remark 1: Composition, Algebraic sum, product and the pointwise limit of al.sl.s.c functions is not in general al.sl.s.c. However we can prove the following:

Theorem 3.7: The uniform limit of a sequence of al.sl.s.c functions is al.sl.s.c.

Note 3: Pasting Lemma is not true for al.sl.s.c functions. However we have the following weaker versions.

Theorem 3.8: Let X and Y be topological spaces such that X = A∪ B and let $f_{/A}$ and $g_{/B}$ are al.sl.r.c maps such that f(x) = g(x) for all x∈A∩B. If A, B∈RO(X) and RO(X) is closed under finite unions, then the combination α: X→ Y is al.sl.s.c continuous.

Theorem 3.9: Pasting Lemma Let X and Y be spaces such that X = A∪B and let $f_{/A}$ and $g_{/B}$ are al.sl.s.c maps such that f(x) = g(x) for all x∈A∩B. A, B∈RO(X) and SO(X) is closed under finite unions, then the combination α:X→Y is al.sl.s.c.
Proof: Let F∈RCO(Y), then α^{-1}(F) = f^{-1}(F)∪g^{-1}(F), where f^{-1}(F)∈SO(A) and g^{-1}(F)∈SO(B) ⇒ f^{-1}(F); g^{-1}(F)∈SO(X) ⇒ f^{-1}(F)∪g^{-1}(F) = α^{-1}(F)∈SO(X). Hence α:X→Y is al.sl.s.c.

Definition 3.2: A function f is said to be almost somewhat semi-continuous if for U∈RO(σ) and f^{-1}(U) ≠ φ, there exists a non-empty semi-open set V in X such that V⊂ f^{-1}(U).

It is clear that every continuous function is almost somewhat continuous and almost somewhat continuous function is almost somewhat semi-continuous. But the converse is not true.

Example 3.3: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {b}, {a, b}, {b, c}, X}. The function f:(X, τ)→ (X, σ) defined by f(a) = b, f(b) = c and f(c) = a is almost somewhat semi-continuous.

Example 3.4: Let X = {a, b, c}, τ = {φ, {b}, {a, b}, {b, c}, X} and σ = {φ, {a}, {b, c}, X}. The function f:(X, τ)→ (X, σ) defined by f(a) = b, f(b) = c and f(c) = a is not almost somewhat semi-continuous.

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Note 4: Every almost somewhat semi-continuous function is almost slightly semi-continuous.

Theorem 3.10: If f is almost somewhat semi-continuous and g is continuous, then $g \bullet f$ is almost somewhat semi-continuous.

Corollary 3.3: If f is almost somewhat semi-continuous and g is r-continuous[r-irresolute], then $g \bullet f$ is almost somewhat semi-continuous.

Theorem 3.11: For a surjective function f, the following statements are equivalent:
(i) f is almost somewhat semi-continuous.
(ii) If C is a r-closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper semi-closed subset D of X such that $f^{-1}(C) \subset D$.
(iii) If M is a dense subset of X, then $f(M)$ is a dense subset of Y.
Proof: (i) ⇒(ii): For C, r-closed in Y such that $f^{-1}(C) \neq X$, Y-C is r-open in Y such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists a semi-open set V such that $V \neq \varphi$ and $V \subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. Thus $X\text{-}V \supset f^{-1}(C)$ and X - V = D is a proper semi-closed set in X.
(ii) ⇒(i): Let $U \in RO(\sigma)$ and $f^{-1}(U) \neq \varphi$ Then Y-U is r-closed and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists a proper semi-closed set D such that $D \supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is semi-open and $X\text{-}D \neq \varphi$.
(ii) ⇒(iii): Let M be dense set in X. If $f(M)$ is not dense in Y. Then there exists a proper r-closed set C in Y such that $f(M) \subset C \subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper semi-closed set D such that $M \subset f^{-1}(C) \subset D \subset X$. This is a contradiction to the fact that M is dense in X.
(iii) ⇒(ii): Suppose (ii) is not true, there exists a r-closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper semi-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C.

Theorem 3.12: Let f be a function and $X = A \cup B$, where $A,B \in RO(X)$. If $f_{/A}$ and $f_{/B}$ are almost somewhat semi-continuous, then f is almost somewhat semi-continuous.
Proof: Let $U \in RO(\sigma)$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is almost somewhat semi-continuous, there exists a semi-open set V in A such that $V \neq \varphi$ and $V \subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since $V \in SO(A)$ and $A \in RO(X)$, $V \in SO(X)$. Thus f is almost somewhat semi-continuous.
The proof of other cases are similar.

Definition 3.3: If X is a set and τ and σ are topologies on X, then τ is said to be semi-equivalent to σ provided if $U \in SO(\tau)$ and $U \neq \varphi$, there is an semi-open set V in X such that $V \neq \varphi$ and $V \subset U$ and if $U \in SO(\sigma)$ and $U \neq \varphi$, there is an semi-open set V in (X, τ) such that $V \neq \varphi$ and $U \supset V$.

Definition 3.4: $A \subset X$ is said to be dense in X if there is no proper closed set C in X such that $M \subset C \subset X$.

Now, consider the identity function f and assume that τ and σ are equivalent. Then f and f^{-1} are almost somewhat continuous. Conversely, if the identity function f is almost somewhat continuous in both directions, then τ and σ are equivalent.

Theorem 3.13: Let $f:(X, \tau) \to (Y, \sigma)$ be a almost somewhat semi-continuous surjection and τ^* be a topology for X, which is semi-equivalent to τ. Then $f:(X, \tau^*) \to (Y, \sigma)$ is almost somewhat semi-continuous.
Proof: Let $V \in RO(\sigma) \ni f^{-1}(V) \neq \varphi$. Since f is almost somewhat semi-continuous, $\exists$ a nonempty $U \in SO(X, \tau) \ni U \subset f^{-1}(V)$. For τ^* is semi-equivalent to τ, $\exists\ U^* \in SO(X; \tau^*) \ni U^* \subset U$. But $U \subset f^{-1}(V)$. Then $U^* \subset f^{-1}(V)$; hence $f:(X, \tau^*) \to (Y, \sigma)$ is almost somewhat semi-continuous.

Theorem 3.14: Let $f:(X, \tau) \to (Y, \sigma)$ be a almost somewhat semi-continuous surjection and σ^* be a topology for Y, which is semi-equivalent to σ. Then $f:(X, \tau) \to (Y, \sigma^*)$ is almost somewhat semi-continuous.
Proof: Let $V^* \in RO(\sigma^*) \ni f^{-1}(V^*) \neq \varphi$. Since σ^* is semi-equivalent to σ, $\exists\ V \neq \varphi \in SO(Y, \sigma) \ni V \subset V^*$. Now $\varphi \neq f^{-1}(V) \subset f^{-1}(V^*)$. Since f is almost somewhat semi-continuous, $\exists\ U \neq \varphi \in SO(X, \tau) \ni U \subset f^{-1}(V)$. Then $U \subset f^{-1}(V^*)$; hence $f:(X, \tau) \to (Y, \sigma^*)$ is almost somewhat semi-continuous.

4. SLIGHTLY SEMI-OPEN MAPPINGS, ALMOST SLIGHTLY SEMI-OPEN MAPPINGS AND ALMOST SOMEWHAT OPEN FUNCTION

Definition 4.1: A function $f: X \to Y$ is said to be
(i) slightly semi-open if image of every clopen set in X is semi-open in Y
(ii) almost slightly semi-open if image of every regular-clopen set in X is semi-open in Y

Note 5:

$$\begin{array}{ccc} \text{slightly-open map} & \to & \text{slightly semi-open.} \\ \downarrow & & \downarrow \\ \text{almost slightly-open map} & \to & \text{almost slightly semi-open.} \end{array}$$

Example 4.1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is slightly open, slightly semi-open, slightly r-open, almost slightly open, almost slightly semi-open and almost slightly r-open.

Example 4.2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X \to Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not slightly open, slightly semi-open, slightly r-open, almost slightly open, almost slightly semi-open and almost slightly r-open.

Note 6:
(i) If $R\alpha O(Y) = SO(Y)$, then f is [almost-]slightly $r\alpha$-open iff f is [almost-]slightly semi-open.
(ii) If $SO(Y) = RO(Y)$, then f is [almost-]slightly-r-open iff f is [almost-]slightly semi-open.
(iii) If $SO(Y) = \alpha O(Y)$, then f is [almost-]slightly α-open iff f is [almost-]slightly semi-open.

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Theorem 4.1: (i) If f is [almost-]slightly open and g is semi-open[r-open] then $g\bullet f$ is slightly semi-open
(ii) If f is [almost-]slightly semi-open and g is M-semi-open[M-r-open] then $g\bullet f$ is slightly semi-open
Proof: Let A be clopen[regular clopen] set in X $\Rightarrow f(A)$ is open in Y $\Rightarrow g(f(A)) = g\bullet f(A)$ is semi-open in Z. Hence $g\bullet f$ is [almost-]slightly semi-open.

Theorem 4.2: If f and g are r-open then $g\bullet f$ is [almost-]slightly semi-open
Proof: Let A be clopen[r-clopen] set in X $\Rightarrow f(A)$ is r-open and so open in Y $\Rightarrow g(f(A))$ is r-open in Z$\Rightarrow g(f(A)) = g\bullet f(A)$ is open in Z. Hence $g\bullet f$ is [almost-]slightly semi-open.

Theorem 4.3: If f is almost slightly-r-open and g is [almost-]semi-open then $g\bullet f$ is [almost-]slightly semi-open

Corollary 4.1:
(i) If f is almost slightly-open and g is open[r-open] then $g\bullet f$ is [almost-]slightly semi-open.
(ii) If f is almost slightly-r-open and g is [almost-]semi-open then $g\bullet f$ is [almost-]slightly semi-open.
(iii) If f and g are almost slightly-r-open then $g\bullet f$ is [almost-]slightly semi-open.

Theorem 4.4: If f is [almost-]slightly semi-open, then $f(A^o) \subset s(f(A))^o$
Proof: Let $A\subset X$ and f is slightly semi-open gives $f(A^o)$ is semi-open in Y and $f(A^o) \subset f(A)$ which in turn gives $f(A^o)^o \subset s(f(A))^o$- (1)
Since $f(A^o)$ is semi-open in Y, $s(f(A^o))^o = f(A^o)$ - - - - - - - - -(2)
From (1) and (2) we have $f(A^o) \subset s(f(A))^o$ for every subset A of X.

Remark 2: converse is not true in general.

Theorem 4.5: If f is slightly semi-open and $A\subset X$ is r-open, then $f(A)$ is τ_s-open in Y.
Proof: Let $A\subset X$ and f is slightly semi-open implies $f(A^o) \subset s(f(A))^o$ which in turn implies $s(f(A))^o \subset f(A)$, since $f(A) = f(A^o)$. But $f(A) \subset s(f(A))^o$. Combining we get $f(A) = s(f(A))^o$. Hence $f(A)$ is τ_s-open in Y.

Corollary 4.2: (i) If f is [almost-]slightly r-open, then $f(A^o) \subset s(f(A))^o$
(ii) If f is [almost-]slightly r-open, then $f(A)$ is τ_s-open in Y if A is r-open set in X.
(iii) If f is almost slightly semi-open and $A\subset X$ is r-open, then $f(A)$ is τ_s-open in Y.

Theorem 4.6: If $s(A)^o = r(A^o)$ for every $A\subset Y$, then the following are equivalent:
(i) f is [almost-]slightly semi-open map
(ii) $f(A^o) \subset s(f(A))^o$
Proof: (i) $\Rightarrow$ (ii) follows from theorem 4.4
(ii) $\Rightarrow$ (i) Let A be any r-open set in X, then $f(A) = s(f(A))^o \supset f(A^o)$ by hypothesis. We have $f(A) \subset s(f(A))^o$. Combining we get $f(A) = s(A)^o = r(A^o)$[by given condition] which implies $f(A)$ is r-open and hence open. Thus f is slightly semi-open.

Theorem 4.7: f is [almost-]slightly semi-open iff for each subset S of Y and each r-clopen set U containing $f^{-1}(S)$, there is a semi-open set V of Y such that $S \subset V$ and $f^{-1}(V) \subset U$.

Remark 3: composition of two [almost-]slightly semi-open maps is not [almost-]slightly semi-open in general

Theorem 4.8: Let X, Y, Z be topological spaces and every open set is r-clopen in Y, then the composition of two [almost-]slightly semi-open maps is [almost-]slightly semi-open.
Proof: Let A be r-clopen in X $\Rightarrow f(A)$ is open and so r-clopen in Y[by assumption] $\Rightarrow g(f(A)) = g\bullet f(A)$ is open in Z. Hence $g\bullet f$ is almost slightly semi-open.

Theorem 4.9: If f is [almost-]slightly g-open; g is open[r-open] and Y is $T_{½}$[r-$T_{½}$], then $g\bullet f$ is [almost-]slightly semi-open.
Proof:(i) Let A be regular clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow f(A)$ is g-open and open in Y[since Y is $T_{½}$] $\Rightarrow g(f(A)) = g\bullet f(A)$ is open in Z. Hence $g\bullet f$ is [almost-]slightly semi-open.

Corollary 4.3: (i) If f is [almost-]slightly g-open; g is open[r-open] and Y is $T_{½}$[r-$T_{½}$] then $g\bullet f$ is [almost-]slightly semi-open.
(ii) If f is [almost-]slightly g-open; g is [almost-]semi-open[almost-r-open] and Y is $T_{½}$[r-$T_{½}$] then $g\bullet f$ is [almost-]slightly semi-open.

Theorem 4.10: If f is [almost-]slightly rg-open; g is open[r-open] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-]slightly semi-open.
Proof: Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow f(A)$ is rg-open and r-open in Y[since Y is r-$T_{½}$] $\Rightarrow g(f(A)) = g\bullet f(A)$ is open in Z. Hence $g\bullet f$ is almost slightly semi-open.

Theorem 4.11: If f is [almost-]slightly rg-open; g is [almost-]semi-open[[almost-]r-open] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-]slightly semi-open.
Proof: Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow f(A)$ is rg-open in Y $\Rightarrow f(A)$ is r-open in Y[since Y is r-$T_{½}$] $\Rightarrow g(f(A)) = g\bullet f(A)$ is open in Z. Hence $g\bullet f$ is almost slightly semi-open.

Corollary 4.4: (i) If f is [almost-]slightly rg-open; g is open[r-open] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-]slightly semi-open.
(ii) If f is [almost-]slightly rg-open; g is [almost-]semi-open[[almost-]r-open] and Y is r-$T_{½}$, then $g\bullet f$ is [almost-]slightly semi-open.

Theorem 4.12: If f; g be two mappings such that $g\bullet f$ is [almost-]slightly semi-open[[almost-] slightly r-open]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-]slightly semi-open
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-]slightly semi-open
(iii) If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-]slightly semi-open
Proof: Let A be regular clopen in Y $\Rightarrow$ A be clopen in Y $\Rightarrow f^{-1}(A)$ is open in X $\Rightarrow g\bullet f(f^{-1}(A)) = g(A)$ is open in Z. Hence g is almost slightly semi-open.

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Similarly we can prove the remaining parts and so omitted.

Corollary 4.5: If f; g be two mappings such that $g \bullet f$ is [almost-]slightly semi-open[[almost-]slightly r-open]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-]slightly semi-open.
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-]slightly semi-open.
(iii) If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-]slightly semi-open.

Theorem 4.13: If X is regular, f is r-open, nearly-continuous, open surjection and $\bar{A} = A$ for every open[r-open] set in Y, then Y is regular.

Theorem 4.14: If f is [almost-]slightly semi-open and A is r-clopen[clopen] set of X, then f_A is [almost-]slightly semi-open.
Proof: Let F be r-open set in A. Then $F = A \cap E$ is r-open in X for some r-open set E of X which implies $f(A)$ is open in Y. But $f(F) = f_A(F)$. Therefore f_A is [almost-]slightly semi-open.

Theorem 4.15: If f is [almost-]slightly semi-open, X is $T_{½}$ and A is g-open set of X, then f_A is [almost-]slightly semi-open.

Corollary 4.6: If f is [almost-]slightly open, X is $T_{½}$ and A is g-open set of X, then f_A is [almost-]slightly semi-open.

Theorem 4.16: If f_i: $X_i \rightarrow Y_i$ be [almost-]slightly semi-open for i = 1, 2. Let f: $X_1\times X_2 \rightarrow Y_1\times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f: $X_1\times X_2 \rightarrow Y_1\times Y_2$ is [almost-]slightly semi-open.
Proof: Let $U_1\times U_2 \subset X_1\times X_2$ where U_i is r-clopen in X_i for i = 1, 2. Then $f(U_1\times U_2) = f_1(U_1) \times f_2(U_2)$ a open set in $Y_1\times Y_2$. Thus $f(U_1\times U_2)$ is open and hence f is [almost-]slightly semi-open.

Corollary 4.7: If f_i: $X_i\rightarrow Y_i$ be [almost-]slightly open for i = 1, 2. Let f: $X_1\times X_2\rightarrow Y_1\times Y_2$ be defined as $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$. Then f:$X_1\times X_2\rightarrow Y_1\times Y_2$ is [almost-]slightly semi-open.

Theorem 4.17: Let h:$X\rightarrow X_1\times X_2$ be [almost-]slightly semi-open. Let f_i: $X\rightarrow X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X\rightarrow X_i$ is [almost-]slightly semi-open for i = 1, 2.
Proof: Let U_1 be r-clopen in X_1, then $U_1 x X_2$ is r-clopen in $X_1 x X_2$, and $h(U_1 x X_2)$ is open in X. But $f_1(U_1) = h(U_1 x X_2)$, therefore f_1 is [almost-]slightly semi-open. Similarly we can show that f_2 is [almost-]slightly semi-open and thus f_i: $X \rightarrow X_i$ is [almost-]slightly semi-open for i = 1, 2.

Corollary 4.8: Let h: $X \rightarrow X_1\times X_2$ be [almost-]slightly open. Let f_i: $X \rightarrow X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \rightarrow X_i$ is [almost-]slightly semi-open for i = 1, 2.

Definition 4.2: A function f is said to be almost somewhat semi-open provided that if $U \in RO(\tau)$ and $U \neq \varphi$, then there exists a non-empty semi-open set V in Y such that $V \subset f(U)$.

Example 4.3: Let X = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {a}, {b, c}, X}. The function f: defined by f(a) = a, f(b) = c and f(c) = b is almost somewhat open and almost somewhat semi-open.

Example 4.4: Let X = {a, b, c}, τ = {φ, {a}, {b, c}, X} and σ = {φ, {a}, {b}, {a, b}, X}. The function f: defined by f(a) = c, f(b) = a and f(c) = b is not almost somewhat semi-open.

Theorem 4.18: Let f be an r-open function and g almost somewhat semi-open. Then $g \bullet f$ is almost somewhat semi-open.

Theorem 4.19: For a bijective function f, the following are equivalent:
(i) f is almost somewhat semi-open.

(ii) If C is an r-closed subset of X, such that $f(C) \neq Y$, then there is a semi-closed subset D of Y such that $D \neq Y$ and $D \supset f(C)$.
Proof: (i) $\Rightarrow$(ii): Let C be any r-closed subset of X such that $f(C) \neq Y$. Then X-C is r-open in X and X-C $\neq \varphi$. Since f is almost somewhat semi-open, there exists a semi-open set $V \neq \varphi$ in Y such that $V \subset f(X\text{-}C)$. Put D = Y-V. Clearly D is semi-closed in Y and we claim $D \neq Y$. If D = Y, then $V = \varphi$, which is a contradiction. Since $V \subset f(X\text{-}C)$, $D = Y\text{-}V \supset (Y\text{-} f(X\text{-}C)) = f(C)$.
(ii) $\Rightarrow$(i): For $U \neq \varphi$ an r-open in X, C = X-U is r-closed in X and $f(X\text{-}U) = f(C) = Y\text{-} f(U)$ implies $f(C) \neq Y$. Therefore, by (ii), there is a semi-closed set D of Y such that $D \neq Y$ and $f(C) \subset D$. Clearly V = Y-D is a semi-open set and $V \neq \varphi$. Also, $V = Y\text{-}D \subset Y\text{-} f(C) = Y\text{-} f(X\text{-}U) = f(U)$.

Theorem 4.20: The following statements are equivalent:
(i) f is almost somewhat semi-open.

(ii) If A is a dense subset of Y, then $f^{-1}(A)$ is a dense subset of X.
Proof: (i) $\Rightarrow$(ii): If A is dense set in Y. If $f^{-1}(A)$ is not dense in X, then there exists a r-closed set B in X such that $f^{-1}(A) \subset B \subset X$. Since f is almost somewhat semi-open and X-B is open, there exists a nonempty semi-open set C in Y such that $C \subset f(X\text{-}B)$. Therefore, $C \subset f(X\text{-}B) \subset f(f^{-1}(Y\text{-}A)) \subset Y\text{-}A$. That is, $A \subset Y\text{-}C \subset Y$. Now, Y-C is a semi-closed set and $A \subset Y\text{-}C \subset Y$. This implies that A is not a dense set in Y, which is a contradiction. Therefore, $f^{-1}(A)$ is a dense set in X.
(ii) $\Rightarrow$(i): If $A \neq \varphi$ is an r-open set in X. We want to show that $(f(A))^o \neq \varphi$. Suppose $(f(A))^o = \varphi$. Then, $cl(f(A)) = Y$. By (ii), $f^{-1}(Y\text{-} f(A))$ is dense in X. But $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$. Now, X-A is r-closed. Therefore, $f^{-1}(Y\text{-} f(A)) \subset X\text{-}A$ gives $X = cl(f^{-1}(Y\text{-} f(A))) \subset X\text{-}A$. This implies that $A = \varphi$, which is contrary to $A \neq \varphi$. Therefore, $(f(A))^o \neq \varphi$. Hence f is almost somewhat semi-open.

Theorem 4.21: Let f be almost somewhat semi-open and A be any r-open subset of X. Then $f_{/A}$:(A; $\tau_{/A}$) $\rightarrow$ (Y, σ) is almost somewhat semi-open.
Proof: Let $U \in RO(\tau_{/A})$ such that $U \neq \varphi$. Since $U \in RO(\tau_{/A})$; $A \in RO(X)$; $U \in RO(X)$ and f is almost somewhat semi-open, $\exists$ $V \in SO(Y)$, such that $V \subset f(U)$. Thus $f_{/A}$ is almost somewhat semi-open.

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Theorem 4.22: Let f be a function and $X = A\cup B$, where $A,B\in\tau(X)$. If the restriction functions $f_{/A}$ and $f_{/B}$ are almost somewhat semi-open, then f is almost somewhat semi-open.
Proof: Let U be any r-open subset of X such that $U\neq \varphi$. Since $X = A\cup B$, either $A\cap U \neq \varphi$ or $B\cap U \neq \varphi$ or both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Since U is open in X, U is open in both A and B.
Case (i): If $A\cap U \neq \varphi$, where $U\cap A\in RO(\tau_{/A})$. Since $f_{/A}$ is almost somewhat semi-open, $\exists$ $V\in SO(Y)$ such that $V\subset f(U\cap A) \subset f(U)$, which implies that f is almost somewhat semi-open.
Case (ii): If $B\cap U \neq \varphi$, where $U\cap B\in RO(\tau_{/B})$. Since $f_{/B}$ is almost somewhat semi-open, $\exists$ $V\in SO(Y)$ such that $V\subset f(U\cap B) \subset f(U)$, which implies that f is almost somewhat semi-open.
Case (iii): If both $A\cap U \neq \varphi$ and $B\cap U \neq \varphi$. Then by cases (i) and (ii) f is almost somewhat semi-open.

Remark 4: Two topologies τ and σ for X are said to be semi-equivalent if and only if the identity function f: $(X, \tau)\rightarrow (Y, \sigma)$ is almost somewhat semi-open in both directions.

Theorem 4.23: Let f: $(X, \tau)\rightarrow (Y, \sigma)$ be a almost somewhat almost semi-open function. Let τ^* and σ^* be topologies for X and Y, respectively such that τ^* is semi-equivalent to τ and σ^* is semi-equivalent to σ. Then f: $(X; \tau^*)\rightarrow (Y; \sigma^*)$ is almost somewhat semi-open.

5. SLIGHTLY SEMI-CLOSED MAPPINGS AND ALMOST SLIGHTLY SEMI-CLOSED MAPPINGS

Definition 5.1: A function $f: X\rightarrow Y$ is said to be
(i) slightly semi-closed if image of every clopen set in X is semi-closed in Y
(ii) almost slightly semi-closed if image of every regular-clopen set in X is semi-closed in Y

Note 7:
slightly-closed map $\rightarrow$ slightly semi-closed.
$\downarrow$ $\downarrow$
almost slightly-closed map$\rightarrow$ almost slightly semi-closed.

Example 5.1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is slightly closed, slightly semi-closed, slightly r-closed, almost slightly closed, almost slightly semi-closed and almost slightly r-closed.

Example 5.2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X\rightarrow Y$ be defined $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is not slightly closed, slightly semi-closed, slightly r-closed, almost slightly closed, almost slightly semi-closed and almost slightly r-closed.

Note 8:
(i) If $R\alpha C(Y) = SC(Y)$, then f is [almost-]slightly $r\alpha$-closed iff f is [almost-]slightly semi-closed.
(ii)If $SC(Y) = RC(Y)$, then f is [almost-]slightly-r-closed iff f is [almost-]slightly semi-closed.
(iii) If $SC(Y) = \alpha C(Y)$, then f is [almost-]slightly α-closed iff f is [almost-]slightly semi-closed.

Theorem 5.1: (i) If f is [almost-]slightly closed and g is semi-closed[r-closed] then $g\bullet f$ is slightly semi-closed
(ii) If f is [almost-]slightly semi-closed and g is M-semi-closed[M-r-closed] then $g\bullet f$ is slightly semi-closed
Proof: Let A be clopen[regular clopen] set in X $\Rightarrow$ $f(A)$ is closed in Y $\Rightarrow$ $g(f(A)) = g\bullet f(A)$ is semi-closed in Z. Hence $g\bullet f$ is [almost-]slightly semi-closed.

Theorem 5.2: If f and g are r-closed then $g\bullet f$ is [almost-]slightly semi-closed
Proof: Let A be clopen[r-clopen] set in X $\Rightarrow$ $f(A)$ is r-closed and so closed in Y $\Rightarrow$ $g(f(A))$ is r-closed in Z$\Rightarrow$ $g(f(A)) = g\bullet f(A)$ is closed in Z. Hence $g\bullet f$ is [almost-]slightly semi-closed.

Theorem 5.3: If f is almost slightly-r-closed and g is [almost-]semi-closed then $g\bullet f$ is [almost-]slightly semi-closed

Corollary 5.1:
(i) If f is almost slightly-closed and g is closed[r-closed] then $g\bullet f$ is [almost-]slightly semi-closed.
(ii) If f and g are almost slightly-r-closed then $g\bullet f$ is [almost-]slightly semi-closed.
(iii) If f is almost slightly-r-closed and g is [almost-]semi-closed then $g\bullet f$ is [almost-]slightly semi-closed.

Theorem 5.4: If f is [almost-]slightly semi-closed, then $scl(f(A))\subset f(cl(A))$
Proof: Let $A\subset X$ and f is slightly semi-closed gives $f(cl(A))$ is semi-closed in Y and $f(A) \subset f(cl(A))$ which in turn gives $scl(f(A)) \subset scl(f(cl(A)))$- - - - - - - - - (1)
Since $f(cl(A))$ is semi-closed in Y, $scl(f(cl(A))) = f(cl(A))$ - - - - - - -(2)
From (1) and (2) we have $(scl(f(A))) \subset (f(cl(A)))$ for every subset A of X.

Remark 5: converse is not true in general.

Theorem 5.5: If f is slightly semi-closed and $A\subset X$ is r-closed, then $f(A)$ is τ_s-closed in Y.
Proof: Let $A\subset X$ and f is slightly semi-closed implies $(scl(f(A))) \subset f(cl(A))$ which in turn implies $(scl(f(A)))\subset f(A)$, since $f(A) = f(cl(A))$. But $f(A)\subset(scl(f(A)))$. Combining we get $f(A) = (scl(f(A)))$. Hence $f(A)$ is τ_s-closed in Y.

Corollary 5.2: (i) If f is [almost-]slightly r-closed, then $scl((f(A)))\subset f(cl(A))$
(ii) If f is [almost-]slightly r-closed, then $f(A)$ is closed in Y if A is r-closed set in X.
(iii) If f is almost slightly semi-closed and $A\subset X$ is r-closed, then $f(A)$ is τ_s-closed in Y.

Theorem 5.6: If $(scl(A)) = r(cl(A))$ for every $A\subset Y$, then the following are equivalent:
(i) f is [almost-]slightly semi-closed map

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(ii) scl(f(A)) $\subset$ f(cl(A))

Proof: (i) $\Rightarrow$ (ii) follows from theorem 5.4

(ii) $\Rightarrow$ (i) Let A be any r-closed set in X, then f(A) = f(cl(A)) $\supset$ (scl(f(A))) by hypothesis. We have f(A) $\subset$ (scl(f(A))). Combining we get f(A) = (scl(f(A))) = r(cl(f(A)))[by given condition] which implies f(A) is r-closed and hence closed. Thus f is slightly semi-closed.

Theorem 5.7: f is [almost-]slightly semi-closed iff for each subset S of Y and each r-clopen set U containing f^{-1}(S), there is a semi-closed set V of Y such that S $\subset$ V and f^{-1}(V) $\subset$ U.

Remark 6: composition of two [almost-]slightly semi-closed maps is not [almost-]slightly semi-closed in general

Theorem 5.8: Let X, Y, Z be topological spaces and every closed set is r-clopen in Y, then the composition of two [almost-]slightly semi-closed maps is [almost-]slightly semi-closed.
Proof: Let A be r-clopen in X $\Rightarrow$ f(A) is closed and so r-clopen in Y[by assumption] $\Rightarrow$ g(f(A)) = g•f(A) is closed in Z. Hence g•f is almost slightly semi-closed.

Theorem 5.9: If f is [almost-]slightly g-closed; g is closed[r-closed] and Y is $T_{½}$[r-$T_{½}$], then g•f is [almost-]slightly semi-closed.
Proof:(i) Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow$ f(A) is g-closed in Y $\Rightarrow$ f(A) is closed in Y[since Y is $T_{½}$] $\Rightarrow$ g(f(A)) = g•f(A) is closed in Z. Hence g•f is [almost-]slightly semi-closed.

Corollary 5.3: (i) If f is [almost-]slightly g-closed; g is closed[r-closed] and Y is $T_{½}$[r-$T_{½}$] then g•f is [almost-]slightly semi-closed.
(ii) If f is [almost-]slightly g-closed; g is [almost-]semi-closed[[almost-]r-closed] and Y is $T_{½}$[r-$T_{½}$] then g•f is [almost-]slightly semi-closed.

Theorem 5.10: If f is [almost-]slightly rg-closed; g is closed[r-closed] and Y is r-$T_{½}$, then g•f is [almost-]slightly semi-closed.
Proof: Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow$ f(A) is rg-closed and so r-closed in Y[since Y is r-$T_{½}$] $\Rightarrow$ g(f(A)) = g•f(A) is closed in Z. Hence g•f is almost slightly semi-closed.

Theorem 5.11: If f is [almost-]slightly rg-closed; g is [almost-]semi-closed[[almost-]r-closed] and Y is r-$T_{½}$, then g•f is [almost-]slightly semi-closed.
Proof: Let A be r-clopen in X $\Rightarrow$ A be clopen in X $\Rightarrow$ f(A) is rg-closed and so r-closed in Y[since Y is r-$T_{½}$] $\Rightarrow$ g(f(A)) = g•f(A) is closed in Z. Hence g•f is almost slightly semi-closed.

Corollary 5.4: (i) If f is [almost-]slightly rg-closed; g is closed[r-closed] and Y is r-$T_{½}$, then g•f is [almost-]slightly semi-closed.
(ii) If f is [almost-]slightly rg-closed; g is [almost-]semi-closed[[almost-]r-closed] and Y is r-$T_{½}$, then g•f is [almost-]slightly semi-closed.

Theorem 5.12: If f; g be two mappings such that g•f is [almost-]slightly semi-closed[[almost-] slightly r-closed]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-]slightly semi-closed
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-]slightly semi-closed
(iii)If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-]slightly semi-closed
Proof: Let A be regular clopen in Y $\Rightarrow$ A be clopen in Y $\Rightarrow$ f^{-1}(A) is closed in X $\Rightarrow$ g•f(f^{-1}(A)) = g(A) is closed in Z. Hence g is almost slightly semi-closed.
Similarly we can prove the remaining parts and so omitted.

Corollary 5.5: If f; g be two mappings such that g•f is [almost-]slightly semi-closed[[almost-]slightly r-closed]. Then the following are true
(i) If f is continuous[r-continuous] and surjective, then g is [almost-]slightly semi-closed.
(ii) If f is g-continuous, surjective and X is $T_{½}$, then g is [almost-]slightly semi-closed.
(iii) If f is rg-continuous, surjective and X is r-$T_{½}$, then g is [almost-]slightly semi-closed.

Theorem 5.13: If X is regular, f is r-closed, nearly-continuous, closed surjection and $\bar{A}$ = A for every closed[r-closed] set in Y, then Y is regular.

Theorem 5.14: If f is [almost-]slightly semi-closed and A is r-clopen[clopen] set of X, then f_A is [almost-]slightly semi-closed.
Proof: For F, r-closed in A, Then F = A$\cap$ E is r-closed in X for some r-closed set E of X which implies f(A) is closed in Y. But f(F) = f_A(F). Therefore f_A is [almost-]slightly semi-closed.

Theorem 5.15: If f is [almost-]slightly semi-closed, X is $T_{½}$ and A is g-closed set of X, then f_A is [almost-]slightly semi-closed.

Corollary 5.6: If f is [almost-]slightly-closed, X is $T_{½}$ and A is g-closed set of X, then f_A is [almost-]slightly semi-closed.

Theorem 5.16: If f_i: $X_i \rightarrow Y_i$ be [almost-]slightly semi-closed for i = 1, 2. Let f:$X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as f(x_1, x_2) = (f_1(x_1), f_2(x_2)). Then f:$X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is [almost-]slightly semi-closed.
Proof: Let $U_1 \times U_2 \subset X_1 \times X_2$ where $U_i \in RCO(X_i)$ for i = 1, 2. Then $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$ a closed set in $Y_1 \times Y_2$. Thus $f(U_1 \times U_2)$ is closed and hence f is [almost-]slightly semi-closed.

Corollary 5.7: If f_i:$X_i \rightarrow Y_i$ be [almost-]slightly semi-closed for i = 1, 2. Let f:$X_1 \times X_2 \rightarrow Y_1 \times Y_2$ be defined as f(x_1, x_2) = (f_1(x_1), f_2(x_2)). Then f:$X_1 \times X_2 \rightarrow Y_1 \times Y_2$ is [almost-]slightly semi-closed.

Theorem 5.17: Let h:$X \rightarrow X_1 \times X_2$ be [almost-]slightly semi-closed. Let f_i: $X \rightarrow X_i$ be defined as h(x) = (x_1, x_2) and f_i(x) = x_i. Then f_i: $X \rightarrow X_i$ is [almost-]slightly semi-closed for i = 1, 2.

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Proof: Let U_1 be r-clopen in X_1, then U_1xX_2 is r-clopen in X_1xX_2, and $h(U_1xX_2)$ is closed in X. But $f_1(U_1) = h(U_1xX_2)$, therefore f_1 is [almost-]slightly semi-closed. Similarly we can show that f_2 is [almost-]slightly semi-closed and thus f_i: $X \to X_i$ is [almost-]slightly semi-closed for i = 1, 2.

Corollary 5.8: Let h: $X \to X_1 \times X_2$ be [almost-]slightly semi-closed. Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is [almost-]slightly semi-closed for i = 1, 2.

6. COVERING AND SEPARATION PROPERTIES OF al.sl.s.c. AND al.swt.s.c. FUNCTIONS

Theorem 6.1: If f is al.sl.s.c.[al.sl.r.c] surjection and X is semi-compact, then Y is compact.
Proof: Let $\{G_i : i \in I\}$ be any r-clopen cover for Y. Then each G_i is r-clopen in Y and f is al.sl.s.c., $f^{-1}(G_i)$ is semi-open in X. Thus $\{f^{-1}(G_i)\}$ forms a semi-open cover for X with a finite subcover, since X is semi-compact. Since f is surjection, $Y = f(X) = \cup^{n}_{i=1} G_i$. Therefore Y is compact.

Theorem 6.2: If f is al.sl.s.c., surjection and X is semi-compact[semi-Lindeloff] then Y is mildly compact[mildly lindeloff].
Proof: Let $\{U_i : i \in I\}$ be r-clopen cover for Y. For each x in X, $\exists\ \alpha_x \in I$ such that $f(x) \in U_{\alpha x}$ and $\exists\ V_x \in SO(X, x) \ni f(V_x) \subset U_{\alpha x}$. Since $\{V_i : i \in I\}$ is a semi-open cover of X, $\exists$ a finite subset I_0 of I such that $X \subset \{V_x : x \in I_0\}$. Thus $Y \subset \cup\{f(V_x) : x \in I_0\} \subset \cup\{U_{\alpha x} : x \in I_0\}$. Hence Y is mildly compact.
Corollary 6.1: (i) If f is al.sl.r.c. surjection and X is semi-compact, then Y is compact.
(ii) If f is al.sl.s.c.[resp**:** al.sl.r.c] surjection and X is locally semi-compact{resp**:** semi-Lindeloff; locally semi-Lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff; locally mildly compact; locally mildly lindeloff}.
(iii)If f is al.sl.s.c., [resp**:** al.sl.r.c] surjection and X is semi-compact[semi-lindeloff] then Y is mildly compact[mildly lindeloff].

Theorem 6.3: If f is al.sl.s.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
Proof: Let $\{V_i : V_i \in RCO(Y); i \in I\}$ be a cover of Y, then $\{f^{-1}(V_i) : i \in I\}$ is semi-open cover of X and so there is finite subset I_0 of I, such that $\{f^{-1}(V_i) : i \in I_0\}$ covers X. Therefore $\{V_i : i \in I_0\}$ covers Y since f is surjection. Hence Y is mildly compact.

Theorem 6.4: If f is al.sl.s.c.,[resp**:** al.sl.r.c.] surjection and X is semi-connected, then Y is connected.
Proof: If Y is disconnected, then $Y = A \cup B$ where A and B are disjoint r-clopen sets in Y. Since f is al.sl.s.c. surjection, $X = f^{-1}(Y) = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ $f^{-1}(B)$ are disjoint semi-open sets in X, which is a contradiction for X is semi-connected. Hence Y is connected.

Corollary 6.2: (i) If f is al.sl.s.c[resp**:** al.sl.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
(ii)The inverse image of a disconnected space under a al.sl.s.c.,[resp**:** al.sl.r.c.] surjection is semi-disconnected.

Theorem 6.5: If f is al.sl.s.c.[resp**:** al.sl.r.c.], injection and Y is UrT_i, then X is sT_i i = 0, 1, 2.
Proof: Let $x_1 \neq x_2 \in X$. Then $f(x_1) \neq f(x_2) \in Y$ since f is injective. For Y is UrT_2 $\exists\ V_j \in RCO(Y)$ such that $f(x_j) \in V_j$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 3.1, $x_j \in f^{-1}(V_j) \in SO(X)$ for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is sT_2.

Theorem 6.6: If f is al.sl.s.c.[al.sl.r.c.] injection; r-closed and Y is UrT_i, then X is sT_i i = 3, 4.
Proof:(i) Let x in X and F be disjoint r-closed subset of X not containing x, then $f(x)$ and $f(F)$ be disjoint r-closed subset of Y not containing $f(x)$, since f is r-closed and injection. Since Y is ultraregular, $f(x)$ and $f(F)$ are separated by disjoint r-clopen sets U and V respectively. Hence $x \in f^{-1}(U)$; $F \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in SO(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is sT_3.
(ii) Let F_j and $f(F_j)$ are disjoint r-closed sets in X and Y respectively for j = 1,2, since f is r-closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint r-clopen sets V_j respectively for j = 1,2. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in SO(X)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is sT_4.

Theorem 6.7: If f is al.sl.s.c.[resp**:** al.sl.r.c.], injection and
(i) Y is UrC_i[resp**:** UrD_i] then X is sC_i[resp**:** sD_i] i = 0, 1, 2.
(ii)Y is UrR_i, then X is sR_i i = 0, 1.

Theorem 6.8: If f is al.sl.s.c.[al.sl.r.c] and Y is UrT_2, then the graph G(f) is semi-closed in X×Y.
Proof: Let $(x_1, x_2) \notin G(f)$ implies $y \neq f(x)$ implies $\exists$ disjoint V; $W \in RCO(Y)$ such that $f(x) \in V$ and $y \in W$. Since f is al.sl.s.c., $\exists$ $U \in SO(X)$ such that $x \in U$ and $f(U) \subset W$ and $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G($f$) is semi-closed in X×Y.

Theorem 6.9: If f is al.sl.s.c.[al.sl.r.c] and Y is UrT_2, then $A = \{(x_1, x_2) | f(x_1) = f(x_2)\}$ is semi-closed in X×X.
Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies $\exists$ disjoint $V_j \in RCO(Y)$ such that $f(x_j) \in V_j$, and since f is al.sl.s.c., $f^{-1}(V_j) \in SO(X, x_j)$ for j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in SO(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is semi-closed.

Theorem 6.10: If f is al.sl.r.c.[resp**:** al.sl.s.c.]; g**:** $X \to Y$ is al.sl.c[resp**:** al.sl.r.c]; and Y is UrT_2, then E = {x in X **:** $f(x) = g(x)$} is semi-closed in X.
We have the following consequences of theorems 6.1 to 6.10:

Theorem 6.11: If f is al.swt.s.c.[resp**:** al.swt.r.c] surjection and X is semi-compact, then Y is compact.

Theorem 6.12: If f is al.swt.s.c., surjection and X is semi-compact[semi-Lindeloff] then Y is mildly compact[mildly lindeloff].

Corollary 6.3: (i) If f is al.swt.r.c. surjection and X is semi-compact, then Y is compact.
(ii) If f is al.swt.s.c.[resp**:** al.swt.r.c] surjection and X is semi-compact[semi-Lindeloff] then Y is mildly compact[mildly lindeloff].

(iii) If f is al.swt.s.c.[resp**:** al.swt.r.c] surjection and X is locally semi-compact{resp**:** semi-Lindeloff; locally semi-Lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff; locally mildly compact; locally mildly lindeloff }.

Theorem 6.13: If f is al.swt.s.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

Theorem 6.14: If f is al.swt.s.c.,[al.swt.r.c.] surjection and X is semi-connected, then Y is connected.

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Corollary 6.4: (i) If *f* is al.swt.c[resp: al.swt.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
(ii)The inverse image of a disconnected space under an al.swt.s.c.,[resp: al.swt.r.c.;] surjection is semi-disconnected.

Theorem 6.15: (i) If *f* is al.swt.s.c.[al.swt.r.c.], injection and Y is UrT_i, then X is sT_i i = 0, 1, 2.
(ii) If *f* is al.swt.s.c.[resp: al.swt.r.c.] injection; r-closed and Y is UrT_i, then X is sT_i i = 3, 4.

Theorem 6.16: If *f* is al.swt.s.c.[resp: al.swt.r.c.;], injection and
(i) Y is UrC_i[resp: UrD_i] then X is sC_i[resp: sD_i] i = 0, 1, 2.
(ii) Y is UrR_i, then X is sR_i i = 0, 1.

Theorem 6.17: If *f* is al.swt.s.c.[resp: al.swt.r.c] and Y is UrT_2, then
(i) the graph G(*f*) of *f* is semi-closed in the product space X× Y.
(ii) A = $\{(x_1, x_2) | f(x_1) = f(x_2)\}$ is semi-closed in the product space X× X.

Theorem 6.18: If *f* is al.swt.r.c.[resp: al.swt.s.c.]; *g*: X→ Y is al.swt.c[resp: al.swt.r.c]; and Y is UrT_2, then E = {x in X : *f*(x) = *g*(x)} is semi-closed in X.

7. CONCLUSION

In this paper we introduced the concept of almost slightly semi-continuous functions, almost somewhat semi-continuous functions, somewhat semi-open mappings, slightly semi-open mappings, almost slightly semi-open mappings, slightly semi-closed mappings, almost slightly semi-closed mappings, studied their basic properties and the interrelationship between other such maps.

REFERENCES

1. Abd El-Monsef ME, Eldeeb SN, Mahmoud RA. β−open sets and β−continuous mappings, *Bull. Fac. Sci. Assiut. Chiv. A*. 1983, 12(1), 77-90
2. Andreivic D. β−open sets, *Math. Vestnick*. 1986, 38, 24-32
3. Arse Nagli Uresin, Aynur kerkin, Noiri T. slightly δ-precontinuous funtions, Commen, *Fac. Sci. Univ. Ark. Series* 2007, 56(2), 1-9
4. Arya SP, Bhamini MP. Some weaker forms of semi-continuous functions, *Ganita* 1982, 33(1-2), 124-134
5. Baker CW. Slightly precontinuous funtions, *Acta Math Hung*, 2002, 94(1-6), 45-52
6. Balasubramanian S, Vyjayanthi PAS. Slightly *v*-continuous functions, *JARPM*., 2012, 4(1), 100–112
7. Di.Maio G, Noiri T. on s-closed spaces, *Indian J. Pure and Appl. Math*., (11)226
8. Di.Maio G. A separation axiom weaker than R_0, *Indian J. Pure and Appl. Math*. 1983, 16, 373-375
9. Dunham W. $T_{1/2}$ Spaces, *Kyungpook Math. J*. 1977, 17, 161-169
10. Gentry KR, Hoyle HB. Somewhat continuous functions. *Czechslovak Math.J*.,1971, 21(96), 5-12
11. Maheswari SN, Prasad R. on R_0 spaces, *Portugal Math*., 1975, 34, 213-217
12. Maheswari SN, Prasad R. on s-normal spaces, *Bull. Math. Soc.Sci.R.S.Roumania*, 1978, 22(70), 27
13. Maheswari SN, Prasad R. some new separation axioms, *Ann. Soc.Sci, Bruxelle*, 1975, 89, 395
14. Mashhour AS, Abd El-Monsef ME, El-Deep SN. α-continuous and α-open mappings, *Acta Math Hung*. 1983, 41(3-4), 231-218
15. Mashhour AS, Abd El-Monsef ME, El-Deep SN. on precontinuous and weak precontinuous functions, *Proc.Math.Phy.Soc.Egypt*, 1982, 3, 47-53
16. Noiri T, Chae GI. A Note on slightly semi continuous functions. *Bull.Cal.Math.Soc* 2000, 92(2), 87-92
17. Noiri T. Slightly β−continuous functions, *Internat. J. Math. & Math. Sci*. 2001, 28(8), 469-478
18. Nour TM. Slightly semi continuous functions. *Bull.Cal.Math.Soc* 1995, 87, 187-190
19. Singhal & Singhal. Almost continuous mappings. *Yokohama J.Math*. 1983, 16, 63-73

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Indian journal of engineering, 2013, 5(13), 44-52,