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Pressure drop-flow rate profile of some locally formulated drilling fluids using Bingham plastic and power law rheological models

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ABSTRACT

In rotary drilling, frictional pressure loss is an integral part of drilling hydraulic analysis as huge viscous forces are overcome in the drillstring and annulus, among others. However, these pressure losses mostly occur during mud circulation as a function of flow rate. Therefore, ascertaining the pressure drop-flow rate profile of the drilling mud in drillstring and annulus is fundamental to optimizing pump power rating to avert longevity in the drilling operation. In this paper, the pressure drop-flow rate profile of some locally formulated drilling fluids: water-based and synthetic-based were evaluated in different flow regimes in drill pipe and annulus using Bingham plastic and Power law rheological models. The results obtained show that Power law model best described the rheology of the formulated synthetic-based drilling fluid. Additionally, the results further depict that with turbulent flow, Power law model results in high pressure loss when compared with Bingham plastic model in drill pipe and vice versa in the annulus. However, with laminar flow, the results indicate that Bingham plastic model results in high pressure drop when compared with Power law model both in drill pipe and annulus. Furthermore, the results depict that the pressure drop-flow rate profile of the formulated mud is in consonant with the correlations of the measured (experimental) and modeled rheological data of the formulated drilling fluids.

Keyword: Pressure drop-flow rate profile, Rheological model, Drill pipe, Annulus, Flow regime.

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1. INTRODUCTION

The success of any drilling operation, to a large extent, depends on the proper selection and monitoring of the drilling fluid system. The ability of a fluid to perform a specific function is dependent on its rheological properties. Establishing correlations between rheological parameters and fluid functionality is the first step toward fluid design and optimization for any application. These drilling fluid properties are primarily responsible for the removal of drill cuttings and exerting sufficient hydrostatic pressure on the formation drilled to prevent formation fluid from flowing into the wellbore, etc., but influence drilling process in many other ways. Unsatisfactory fluid performance can lead to such serious problems as bridging the hole, filling the bottom of the hole with drill cuttings, reduced penetration rate, enlargement of the drilled hole, stuck pipe, loss circulation and even a blow out. To a large extent, the successful suspending and lifting of the drilled cuttings to the surface depends on the drilling mud's rheological properties. In rotary drilling, the mud's rheological properties are used to evaluate the frictional pressure drop loss (pressure drop) in the drill pipe as well as the mud flow rate. As such, it is important to ascertain the exact pressure drop along the drill pipe for many reasons. These include, among others: optimizing the pressure drop at the drilling bit in order to have maximum impact force, avoiding fracturing the drilled formation as a result of underestimated annular pressure drop, optimizing the flow rate in the annular in order to achieve effective cuttings transport to the surface (optimum hole cleaning), as well as designing the mud pump available in the drilling rig. In all these cases, the basis is the understanding of the rheological characteristics of the drilling fluid used (Maglione and Robotti, 1996).

Frictional pressure loss is an important part of drilling hydraulic analysis since large viscous forces must be overcome to move drilling fluid through the longer, slender pipe, and annuli used in the drilling process. The success of a drilling fluid can be directly attributed to its viscosity at the shear rate of interest. In practice, fluids are subjected to a wide range of shear rates. Therefore, a thorough understanding of fluid rheology and the impact of shear is necessary to optimize fluid design. Models are developed to describe the shear stress/shear rate relationship of non-Newtonian fluids. These models are used to characterize fluid properties in an effort to determine the ability of a fluid to perform specific functions. In order to optimize fluid performance, an in-depth analysis of rheological models and their inherent limitations is necessary. A rheological model is a mathematical model that is used to describe the relationships between the viscous forces present in the fluid. A rheological model also describes the flow behavior of a fluid by expressing the relationship between the shear rate and the shear stress. Various rheological models are used to describe the behavior of several ideal non-Newtonian fluids and are discussed in another section of this paper. In conventional drilling, drilling fluids are modeled with classical rheological models such as Bingham plastic or Power law model and fluid behavior is defined with only two points of the rheological relations (Simon,

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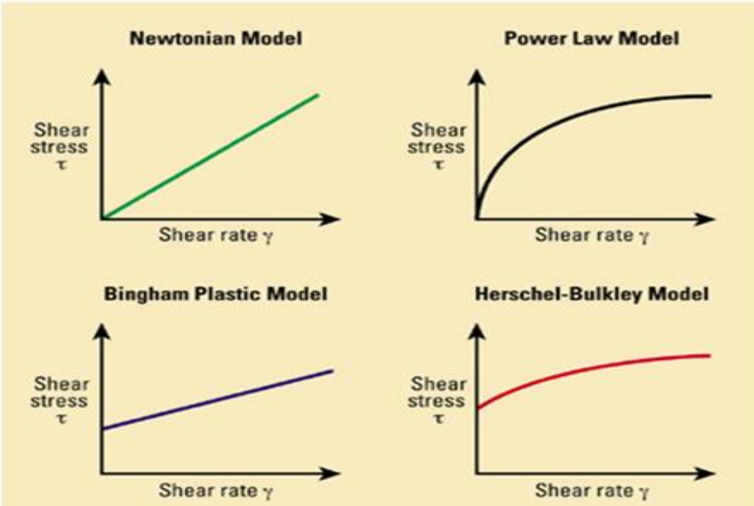

Figure 1

Rheological models (After Hemphill et al., 1993)

2004). The Herschel-Bulkley or Yield Power law model is thought to represent the flow behavior of drilling fluid and is believed to have a comparative advantage over the two models mentioned earlier, as this model incorporates both the theoretical and practical aspects of Bingham plastic and Power law models. However, the complexity in establishing pressure drop-flow rate relationship for the Herschel-Bulkley rheological model is a major concern for many scientists and researchers. As earlier alluded to, knowledge of rheological data and methods of predicting pressure losses are fundamental in evaluating proper pump rate to avoid any longevity in the drilling operation. Generally, when drilling fluid circulates, pressure drop takes place due to friction between the fluid and the surface in contact (Chowdhury et al., 2009). Simon (2004) opined that when drilling fluid behavior deviates from the simple Newtonian flow, frictional pressure loss equation becomes more complex and less accurate due to many simplifying assumptions. The inconsistency in the pressure drop calculation during drilling is due to the rheological model used in the development of the theoretical or empirical correlation. In this connection, due to the complexity in establishing pressure drop-flow rate relationship for the Herschel-Bulkley rheological model, this paper considers two rheological models: Bingham plastic and Power law models in the assessment of the pressure drop-flow rate profile of some locally formulated drilling fluids in both drill pipe and annulus during drilling. Consequently, the ultimate objective of this paper is to standardize the Bingham plastic and Power law rheological models from the locally formulated drilling fluids in the simplest and most accurate way possible and hopefully, make them useful and easily understood tools for the oil and gas industry.

2. BASIC DRILLING MUD RHEOLOGICAL MODELS

A rheological model describes the relationship between shear stress and shear rate when a fluid flows through a circular section or an annulus (Chowdhury et al., 2009). The four basic rheological models considered in drilling mud are presented graphically in Figure 1.

2.1. Newtonian model

This model describes a fluid considering a linear relationship between the shear stress (τ) and shear rate (γ). The equation that describes the shear stress-shear rate relationship is expanded as:

$$\tau = \mu\gamma \tag{1}$$

Graphically, this is represented as a straight line passing through the origin with the slope representing the dynamic viscosity (μ) of the fluid. However, this linearity between shear stress and shear rate is valid as long as the fluid flow regime is laminar (Bourgoyne et al., 2003). Thus, at low shear rate the flow regime is laminar and becomes turbulent at high rates. In this rheological model, the viscosity is constant and is only influenced by changes in temperature and pressure (Rabia, 1985).

2.2. Bingham Plastic Model

It is worth noting that most drilling fluids are too complex to be characterized with a constant viscosity. Thus, the shear stress-shear rate measurement will not exhibit linearity. Therefore, Bingham plastic model presents the decrease of apparent viscosity with increasing shear rate (pseudoplastic) of the drilling fluid (mud). The equation that describes this model is expanded as:

$$\tau = \tau_y + \mu_p\gamma \tag{2}$$

A Bingham plastic will not flow until the applied shear stress (τ) exceeds a certain minimum value (τ_y) known as the yield point (Bourgoyne et al., 2003). When this point is exceeded, changes in shear stress are proportional to changes in shear rate and the proportionality constant is known as plastic viscosity (μ_p).

2.3. Power Law Model

This rheological model is also referred to as Ostwald-de Walle model. Like the Bingham plastic model, this model requires two parameters: the consistency index (K) and the flow behavior index (n) for fluid characterization. The power law model is defined by:

$$\tau = K\gamma^n \tag{3}$$

This can be used to represent a pseudoplastic fluid ($n < 1$), a Newtonian fluid ($n = 1$) and a dilatant fluid ($n > 1$). Therefore, the deviation of the dimensionless flow behavior index (n) from unity characterizes the degree to which the fluid behavior is non-Newtonian. In addition, the power law model equation is valid only for laminar flow regime; thus, at low shear rate.

2.4. Herschel-Bulkley model

This rheological model is a combination of Bingham plastic and Power law models which in some cases, are referred to as Yield Power law model. The equation that expands the rheological model is given as:

$$\tau = \tau_y + K\gamma^n \tag{4}$$

The Herschel-Bulkley equation is preferred to Power law or Bingham plastic relationships because it results in more accurate models of rheological behavior when adequate experimental data are available (Hemphill et al., 1993). However, the model equation is reduced to Bingham plastic model when $n = 1$; where K represents μ_p and Power law model when $\tau_y = 0$. It should

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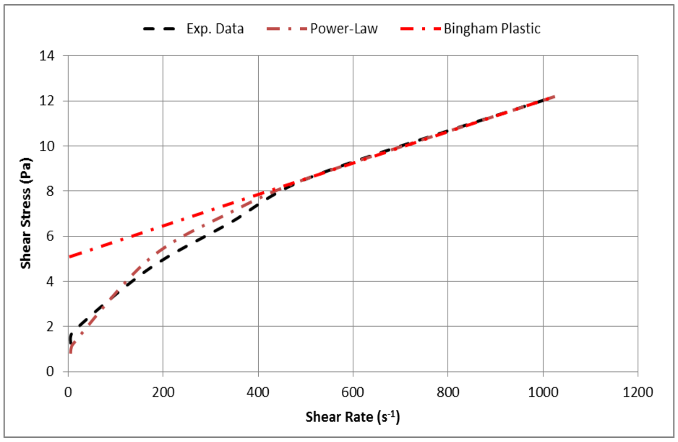

Figure 2
Rheogram (Water-Based mud)

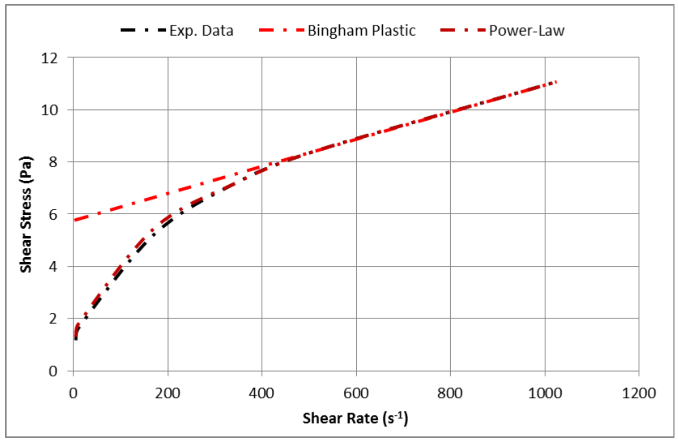

Figure 3
Rheogram (Synthetic-Based mud)

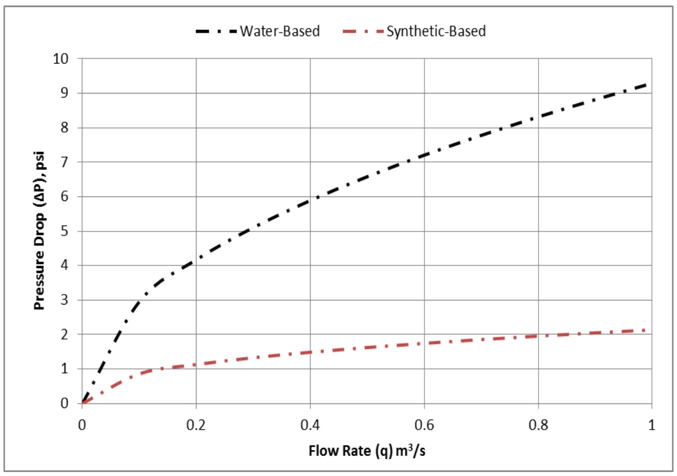

Figure 4
Laminar Flow (pipe) – Power law Model

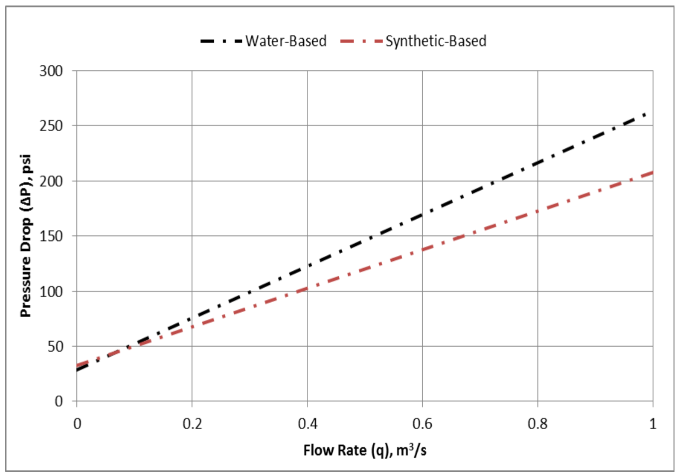

Figure 5
Laminar Flow (pipe) – Bingham plastic Model

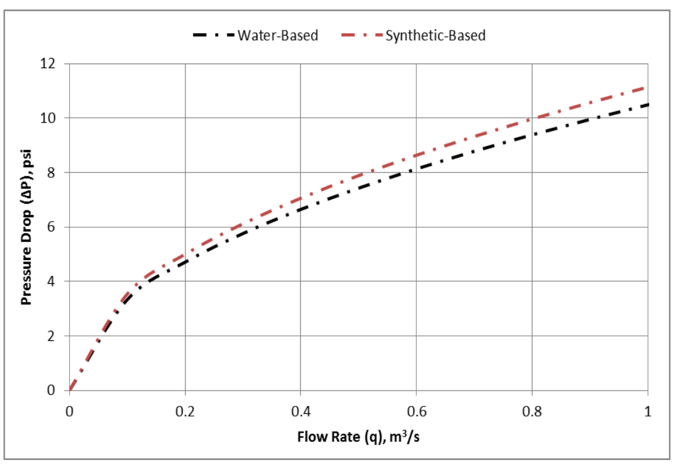

Figure 6
Laminar Flow (annulus) – Power law Model

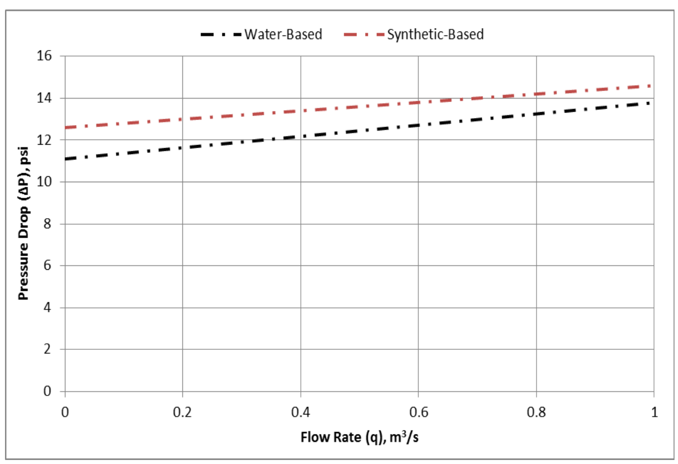

Figure 7
Laminar Flow (annulus) – Bingham plastic Model

be noted that this model can yield mathematical expressions that are not readily solved analytically but can be solved using non-linear regression (Chowdhury et al., 2009).

3. MATERIALS AND METHODS

In this study, two drilling fluid (mud) samples were formulated: water-based and synthetic-based drilling mud. The water-based mud was formulated with bentonite, the clay of choice in drilling operation using American Petroleum Institute (API)

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Table 1
Properties of water-based and synthetic-based mud

Parameters	Water-based	Synthetic-based
Density (ρ), kg/m^3	1031.0	995.0
Plastic Viscosity (μ_{pl}), Pas	0.0070	0.0052
Yield Point (τ_y), Pa	5.0750	5.7600
Flow Behaviour Index (n)	0.4975	0.3952
Consistency Index (K), Pas^{-n}	0.3877	0.7154

Table 2
Mud Rheology Measurements

Rotor Speed (rpm)	Viscometer Dial Reading (degree)	
	Water-Based Mud	Synthetic-Based Mud
3	2.50	3.00
6	3.50	3.50
100	9.00	11.00
200	13.00	14.00
300	17.00	16.00
600	24.00	22.00

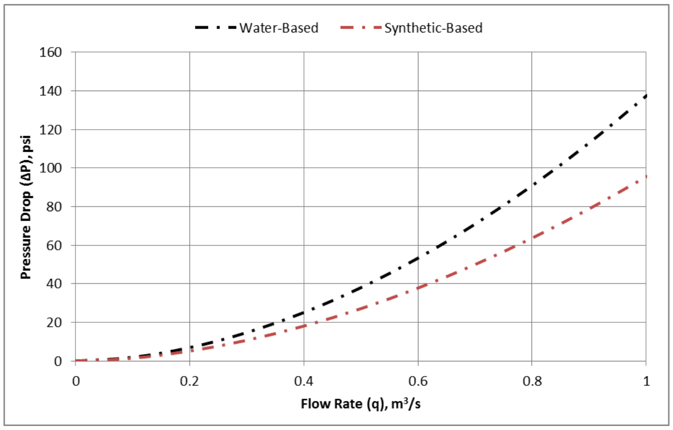

Figure 8
Turbulent Flow (pipe) – Power law Model

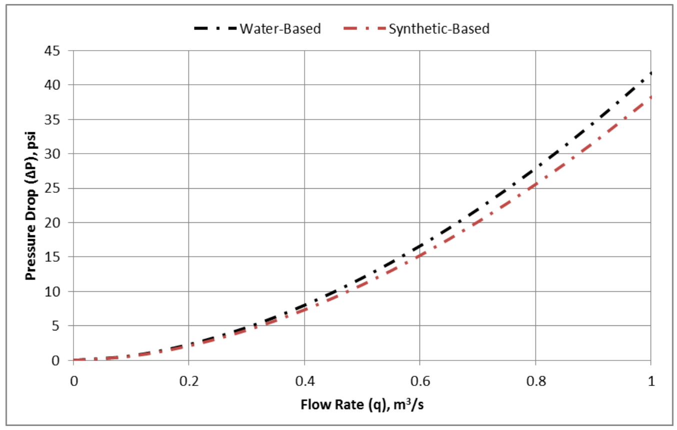

Figure 9
Turbulent Flow (pipe) – Bingham plastic Model

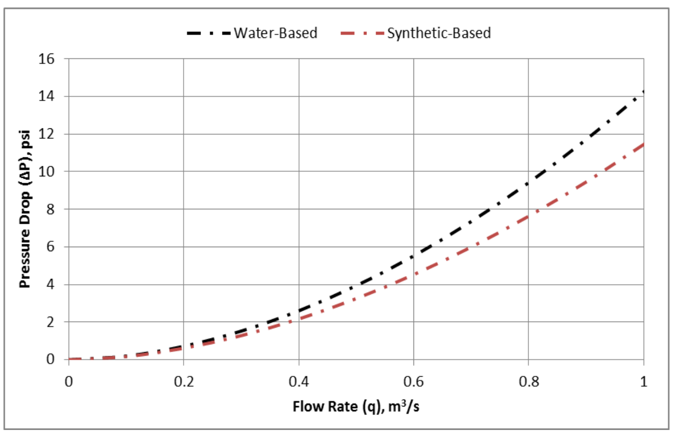

Figure 10
Turbulent Flow (annulus) – Power law Model

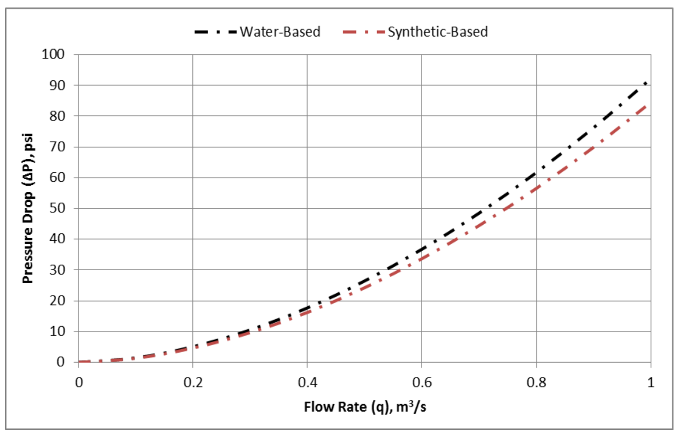

Figure 11
Turbulent Flow (annulus) – Bingham plastic Model

standard of 25g of non-treated bentonite per 350mL of water. The synthetic-based mud was formulated with palm-oil derived ester using about 100g of palm fruit pulp to 350mL of water to obtain the synthetic-based drilling fluid. The basic properties of the formulated water-based mud and the synthetic-based drilling fluid required in this study are presented in Table 1 and these properties are determined based on two data-point standard approach for both Bingham plastic and Power law models. The rheological properties and density of the formulated mud were measured using viscometer and mud balance respectively. The viscometer dial readings obtained and the corresponding rotor speed are presented in Table 2. The obtained experimental rheological data were used to model the rheological regime of the drilling fluids based on an earlier mentioned approach (two data-point) for both Bingham plastic and Power law models. These modeled regimes were compared with the experimental results and are presented in Figures 2 and 3 for both drilling muds: water-based and synthetic-based drilling fluids respectively. To evaluate the pressure drop-flow rate profiles of the formulated drilling fluids in two flow regimes (laminar and turbulent flow) both in drill pipe and annulus, well description was adopted from the work of Chowdhury et al., (2009). The adopted well description is presented in Table 3. Furthermore, the equations that describe

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Table 3
Well Description

Parameter	Dimension
Open Hole Diameter (d_{OH}), m	0.4445
Drill Pipe Outer Diameter (d_o), m	0.1270
Drill Pipe Inner Diameter (d_i), m	0.1087
Well Depth (L), m	789

Table 4
Correlation between Experimental and Modeled Data

Drilling mud	Rheological model	
	Power law	Bingham plastic
Water-based	0.9620	0.6178
Synthetic-based	0.9940	0.6812

pressure drop-flow rate profile in drill pipe and annulus for Bingham plastic and Power law rheological models during laminar and turbulent flow regimes are expanded in equations 5 through 12 in the Appendix. Aside from comparing the rheological regimes between experimental and modeled data, the degree of correlation between them were determined and is presented in Table 4.

4. RESULTS AND DISCUSSION

Evaluation of the pressure drop-flow rate profile of drilling mud is based on the modeled rheological parameters. However, these modeled rheological parameters are contingent upon the consistency of the experimental (measured) data at various shear rates. The determined degree of correlation between the experimental data and the modeled data as presented in Table 4 indicates that Power law model data has the value of 0.9620 and 0.9940 for water-based and synthetic-based drilling fluid respectively. This table also shows Bingham plastic model correlation value of 0.6178 and 0.6812 for water-based mud and synthetic-based drilling fluid respectively. These correlation values go a long way to confirm that in Bingham plastic model, the modeled data overestimate the yield point of the mud rheology as shown in Figures 2 and 3. In addition, Figures 2 and 3 depict the comparison of the experimental data and modeled results from both rheological models: Power law and Bingham plastic. The figures indicate that Power law model regime is positively correlated with the measured (experimental) data for both drilling fluids. In Figure 3, it can be seen that a best fit was obtained with Power law model for the synthetic-based mud, an indication that Power law model can best describe the rheology of the formulated drilling fluid.

It is worth noting that during laminar flow, the flow is dominated by the viscosity of the fluid (mud). Figures 4 through 7 depict the pressure drop-flow rate profile for laminar flow regime in the drill pipe and annulus, where Figures 4 and 5 indicate the profile for the different rheological mud models in drill pipe. With Power law model (Figure 4), the result shows that the formulated synthetic-based fluid has a lower pressure drop in the drill pipe when compared with the water-based mud. However, this was merely the case with Bingham plastic model, as the difference between the mud's profiles was comparable. This is attributed to the comparable plastic viscosity of the drilling fluids as presented in Table 1. Generally, Bingham plastic model overestimates and Power law model underestimates pressure losses (Simon, 2004). This fact was observed in the results depicted in Figures 4 and 5, where the pressure drop (ΔP) for the synthetic-based mud was about 2psi with Power law model and about 200psi with Bingham plastic model at the flow rate of $1.0m^3/s$. This discrepancy with modeled results was also obtained with water-based mud. Figures 6 and 7 depict the laminar flow regime in the annulus. In these figures, it was observed that in both Power law and Bingham plastic models, the pressure drop (ΔP) in the synthetic-based mud was slightly higher than that of the water-based mud. However, mention must be made of the fact that, in both rheological models, the obtained pressure drop (ΔP) in the annulus was comparable; a factor that is attributed to the carrying capacity (yield point) of these formulated drilling muds as indicated in Table 1. It should be noted that turbulent flow is chaotic flow and the velocity varies continuously both in the x and y direction, resulting in high energy consumption and correspondingly high loss due to friction or head loss (Skalle, 2010). Generally, in this flow regime, pressure loss (ΔP) increases exponentially with flow rate as the flow is controlled by the fluid density. Figures 8 through 11 depict the pressure drop-flow rate profile for turbulent flow regime in the drill pipe and annulus of the rheological models with the formulated drilling fluids under study. The results indicate that, with Power law model, high pressure loss was observed with the water-based mud when compared to the synthetic-based drilling fluid. As presented in Table 1, the differences between the formulated mud densities accounted for these observed results. However, Bingham plastic model (Figure 9) results in comparable result with low pressure drop in the drill pipe, when compared with the result obtained with that of Power law model. As noted, the fluid density dominance during turbulent flow regime in drill pipe is significant in Power law model when compared to Bingham plastic model. Furthermore, Figures 10 and 11 present the annulus flow result during turbulent flow regime. The figures indicate that low pressure drop was obtained with Power law model as compared to Bingham plastic model result. Accordingly, the figures further indicate that water-based mud results in high pressure drop with both rheological models when compared to synthetic-based fluid. However, in Bingham plastic model, the formulated mud's pressure-drop profiles are about the same up till $0.4m^3/s$ where there is deviation in their profile.

In summary, the modeled rheological parameters are significant in predicting pressure drop-flow rate profile during flow regime. As observed, in Bingham plastic model, these rheological parameters: plastic viscosity and yield point are the determining factors in drill pipe and annulus during laminar flow regime. However, with turbulent flow regime, plastic viscosity and density become the determining drilling mud parameters to predict pressure drop-flow rate profile. On the other hand, in Power law model, flow behaviour index (n) and consistency index (K) determine the profile during laminar flow in drill pipe and annulus. Thus, mud density, flow behaviour and consistency indexes determine the pressure drop-flow rate profile during turbulent flow regime in both drill pipe and annulus.

5. CONCLUSION

Some locally formulated drilling fluids were tested and their rheological properties were modeled using Bingham plastic and Power law models. In conventional drilling, the increase in equivalent circulating density (ECD) by annular pressure losses is usually small when compared to hydrostatic pressure gradient. Annular pressure losses depend on fluid rheology, flow regime and geometry of the annulus. Application of model has important implications for calculating mud hydraulics in hole drilling

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process because Bingham plastic model overestimates and Power law model underestimates pressure loss in annulus. The accuracy of the mud properties is essential for accurate predictions of ECD in order to obtain a successful drilling operation. Therefore, pressure drop-flow rate profiles provide the essential tool to optimize down-hole equipment and achieve optimum lifting efficiency when circulating at any rate. Based the results of the pressure drop-flow rate profile of the locally formulated drilling fluids, the following conclusions are drawn:

- The shear stress-shear rate relationship of the formulated water-based and synthetic-based drilling fluids can best be described with Power law rheological model.
- Turbulent flow regime in drill pipe and annulus profiles were controlled by mud density, flow behaviour and consistency indexes whereas in laminar flow regime, flow behavior and consistency indexes dominated in drill pipe and annulus profiles when Power law model was considered.

Rheological parameters such as plastic viscosity and yield point in Bingham plastic model dominated in both drill pipe and annulus profiles with laminar flow regime whilst turbulent flow regime was controlled by mud density and plastic viscosity in drill pipe and annulus profiles.

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APPENDIX

Equations:

Bingham Plastic Model

Laminar Flow (drill pipe):

$$\Delta P_{pipe} = A_1 + B_1 q \qquad (5)$$

Where:

$$A_1 = 5.333\left(\frac{L\tau_y}{d_i}\right)$$

$$B_1 = 40.727\left(\frac{\mu_{pl}L}{d_i^4}\right)$$

Laminar Flow (annulus):

$$\Delta P_{annulus} = A_2 + B_2 q \qquad (6)$$

Where:

$$A_2 = \frac{6L\tau_y}{(d_{OH} - d_o)}$$

$$B_2 = 60.091\left[\frac{\mu_{pl}L}{(d_{OH} - d_o)^2(d_{OH}^2 - d_o^2)}\right]$$

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Turbulent Flow (drill pipe):

$$\Delta P_{tur/pipe} = A_3 q^{1.8} \quad (7)$$

Where:

$$A_3 = \frac{0.1128 \rho_m^{0.8} \mu_{pl}^{0.2} L}{d_i^{4.8}}$$

Turbulent Flow (annulus):

$$\Delta P_{tur/ann} = A_4 q^{1.8} \quad (8)$$

Where:

$$A_4 = \frac{0.1128 \rho_m^{0.8} \mu_{pl}^{0.2} L}{(d_{OH} - d_o)^{1.2}(d_{OH}^{3.6} - d_o^{3.6})}$$

Power Law Model

Laminar Flow (drill pipe):

$$\Delta P_{pipe} = C_1^{-n} q^n \quad (9)$$

Where:

$$C_1 = 0.393 \left(\frac{d_i}{4LK}\right)^{1/n} \left[\frac{n}{1+3n}\right] d_i^3$$

Laminar Flow (annulus):

$$\Delta P_{annulus} = A_1 C_2^n q^n \quad (10)$$

Where:

$$A_1 = 4K\left(\frac{L}{d_{OH} - d_o}\right)$$

$$C_2 = \left[\frac{15.273}{(d_{OH} - d_o)(d_{OH}^2 - d_o^2)} \frac{2n+1}{3n}\right]$$

Turbulent Flow (drill pipe):

$$\Delta P_{tur/pipe} = A_2 C_3 q^{A_3} \quad (11)$$

Where:

$$A_2 = \frac{3.242 a L \rho_m}{d_i^5}$$

$$A_3 = 2 + (n-1)b$$

$$C_3 = \left[\frac{d^n 4^{2-n} \rho_m}{K_p 8^{n-1} (\pi d^2)^{2-n}}\right]^{-b}$$

$$K_p = K\left(\frac{3n+1}{4n}\right)^n$$

Turbulent Flow (annulus):

$$\Delta P_{tur/ann} = A_4 C_4 q^{A_3} \quad (12)$$

Where:

$$A_4 = \frac{3.242 a L \rho_m}{(d_{OH} - d_o)(d_{OH}^2 - d_o^2)}$$

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$$C_4 = \left[\frac{(d_{OH} - d_o)^n 4^{2-n} \rho_m}{K_a 12^{n-1}[\pi(d_{OH}^2 - d_o^2)]^{2-n}}\right]^{-b}$$

$$K_a = K\left(\frac{2n+1}{3n}\right)^n$$

A_3 in equation 12 is the same as in equation 11. The constants a and b, in Equations 11 and 12 are expanded as:

$$a = (Log(n) + 3.93)/50$$

$$b = \left(1.75 - Log(n)\right)/7$$

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