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Minimal *rp*-closed sets and Maximal *rp*-closed sets

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ABSTRACT
The object of the present paper is to study the notions of minimal rp-closed set and maximal rp-closed set and their basic properties are studied.

Keywords: rp-closed set and minimal rp-closed set and maximal rp-closed set

1. INTRODUCTION

Nakaoka and Oda have introduced minimal open sets and maximal open sets, which are subclasses of open sets. A. Vadivel and K. Vairamanickam introduced minimal rgα-open sets and maximal rgα-open sets in topological spaces.S. Balasubramanian and P.A.S. Vyjayanthi introduced minimal v-open sets and maximal v-open sets; minimal v-closed sets and maximal v-closed sets in topological spaces. Inspired with these developments we further study a new type of closed namely minimal rp-closed sets and maximal rp-closed sets. Throughout the paper a space X means a topological space (X, τ). The class of sp-closed sets is denoted by spC(X). For any subset A of X its complement, interior, closure, sp-interior, sp-closure are denoted respectively by the symbols A^c, A^o, A^-, $sp(A)^0$ and $sp(A)^-$.

2. PRELIMINARIES

Definition 2.1: A proper nonempty
(i) closed subset U of X is said to be a **minimal closed set** if any closed set contained in U is ϕ or U.
(ii) semi-closed subset U of X is said to be a **minimal semi-closed set** if any semi-closed set contained in U is ϕ or U.
(iii) pre-closed subset U of X is said to be a **minimal pre-closed set** if any pre-closed set contained in U is ϕ or U.
(iv) *v*-closed subset U of X is said to be a **minimal *v*-closed set** if any *v*-closed set contained in U is ϕ or U.
(v) *rg*α-closed subset U of X is said to be a **minimal *rg*α-closed set** if any *rg*α-closed set contained in U is ϕ or U.

Definition 2.2: A proper nonempty
(i) closed subset U of X is said to be a **maximal closed set** if any closed set containing U is X or U.
(ii) semi-closed subset U of X is said to be a **maximal semi-closed set** if any semi-closed set containing U is X or U.
(iii) pre-closed subset U of X is said to be a **maximal pre-closed set** if any pre-closed set containing U is X or U.
(iv) *v*-closed subset U of X is said to be a **maximal *v*-closed set** if any *v*-closed set containing U is X or U.
(v) *rg*α-closed subset U of X is said to be a **maximal *rg*α-closed set** if any *rg*α-closed set containing U is X or U.

3. MINIMAL *RP*-CLOSED SETS AND MAXIMAL *RP*-CLOSED SETS

We now introduce minimal *rp*-closed sets and maximal *rp*-open sets in topological spaces as follows.

Definition 3.1: A proper nonempty *rp*-closed subset F of X is said to be a **minimal *rp*-closed set** if any *rp*-closed set contained in F is ϕ or F.

Remark 1: Minimal closed set and minimal *rp*-closed set are not same:

Example 1: Let X = {a, b, c}; τ = {ϕ, {a, c}, X}. {b} is Minimal closed but not Minimal *rp*-closed set, {a} and {c} are Minimal *rp*-closed but not Minimal closed.

Definition 3.2: A proper nonempty *rp*-open U $\subset$ X is said to be a **maximal *rp*-open set** if any *rp*-open set containing U is either X or U.

Theorem 3.1: A proper nonempty subset U of X is maximal *rp*-open set iff X-U is a minimal *rp*-closed set.
Proof: Let U be a maximal *rp*-open set. Suppose X-U is not a minimal *rp*-closed set. Then $\exists$ *rp*-closed set V $\neq$ X-U such that $\phi \neq$ V $\subset$ X-U. That is U $\subset$ X-V and X-V is a *rp*-open set which is a contradiction for U is a minimal *rp*-closed set. Conversely let X-U be a minimal *rp*-closed set. Suppose U is not a maximal *rp*-open set. Then $\exists$ *rp*-open set E $\neq$ U such that U $\subset$ E $\neq$ X.

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That is $\phi \neq X\text{-}E \subset X\text{-}U$ and X-E is a *rp*-closed set which is a contradiction for X-U is a minimal *rp*-closed set. Therefore U is a maximal *rp*-closed set.

Lemma 3.1:
(i) Let U be a minimal *rp*-closed set and W be a *rp*- closed set. Then $U \cap W = \phi$ or U subset W.
(ii) Let U and V be minimal *rp*- closed sets. Then $U \cap V = \phi$ or $U = V$.
Proof: (i) Let U be a minimal *rp*-closed set and W be a *rp*-closed set. If $U \cap W = \phi$, then there is nothing to prove. If $U \cap W \neq \phi$. Then $U \cap W \subset U$. Since U is minimal *rp*-closed set, we have $U \cap W = U$. Therefore $U \subset W$.
(ii) Let U and V be minimal *rp*-closed sets. If $U \cap V \neq \phi$, then $U \subset V$ and $V \subset U$ by (i). Therefore $U = V$.

Theorem 3.2: Let U be minimal *rp*-closed set. If $x \in U$, then $U \subset W$ for any regular open neighborhood W of x.
Proof: Let U be a minimal *rp*-closed set and x be an element of U. Suppose $\exists$ an regular open neighborhood W of x such that $U \not\subset W$. Then $U \cap W$ is a *rp*-closed set such that $U \cap W \subset U$ and $U \cap W \neq \phi$. Since U is a minimal *rp*-closed set, we have $U \cap W = U$. That is $U \subset W$, which is a contradiction for $U \not\subset W$. Therefore $U \subset W$ for any regular open neighborhood W of x.

Theorem 3.3: Let U be a minimal *rp*-closed set. If $x \in U$, then $U \subset W$ for some *rp*-closed set W containing x.

Theorem 3.4: Let U be a minimal *rp*-closed set. Then $U = \cap\{W : W \in RPO(X, x)\}$ for any element x of U.
Proof: By theorem[3.3] and U is *rp*-closed set containing x, we have $U \subset \cap\{W : W \in RPO(X, x)\} \subset U$.

Theorem 3.5: Let U be a nonempty *rp*-closed set. Then the following three conditions are equivalent.
(i) U is a minimal *rp*-closed set
(ii) $U \subset rp(S)^-$ for any nonempty subset S of U
(iii) $rp(U)^- = rp(S)^-$ for any nonempty subset S of U.
Proof: (i) $\Rightarrow$ (ii) Let $x \in U$; U be minimal *rp*-closed set and $S(\neq \phi) \subset U$. By theorem[3.3], for any *rp*-closed set W containing x, $S \subset U \subset W \Rightarrow S \subset W$. Now $S = S \cap U \subset S \cap W$. Since $S \neq \phi$, $S \cap W \neq \phi$. Since W is any *rp*-closed set containing x, by theorem[3.3], $x \in rp(S)^-$. That is $x \in U \Rightarrow x \in rp(S)^- \Rightarrow U \subset rp(S)^-$ for any nonempty subset S of U.
(ii) $\Rightarrow$ (iii) Let S be a nonempty subset of U. That is $S \subset U \Rightarrow rp(S)^- \subset rp(U)^- \rightarrow$ (1). Again from (ii) $U \subset rp(S)^-$ for any $S(\neq \phi) \subset U \Rightarrow rp(U)^- \subset rp(rp(S)^-)^- = rp(S)^-$. That is $rp(U)^- \subset rp(S)^- \rightarrow$ (2). From (1) and (2), we have $rp(U)^- = rp(S)^-$ for any nonempty subset S of U.
(iii) $\Rightarrow$ (i) From (3) we have $rp(U)^- = rp(S)^-$ for any nonempty subset S of U. Suppose U is not a minimal *rp*-closed set. Then $\exists$ a nonempty *rp*-closed set V such that $V \subset U$ and $V \neq U$. Now $\exists$ an element a in U such that $a \notin V \Rightarrow a \in V^c$. That is $rp(\{a\})^- \subset rp(V^c)^- = V^c$, as V^c is *rp*-closed set in X. It follows that $rp(\{a\})^- \neq rp(U)^-$. This is a contradiction for $rp(\{a\})^- = rp(U)^-$ for any $\{a\}(\neq \phi) \subset U$. Therefore U is a minimal *rp*-closed set.

Theorem 3.6: If $V \neq \phi$ finite *rp*-closed set. Then $\exists$ at least one (finite) minimal *rp*-closed set U such that $U \subset V$.
Proof: Let V be a nonempty finite *rp*-closed set. If V is a minimal *rp*-closed set, we may set U = V. If V is not a minimal *rp*-closed set, then $\exists$ (finite) *rp*-closed set V_1 such that $\phi \neq V_1 \subset V$. If V_1 is a minimal *rp*-closed set, we may set $U = V_1$. If V_1 is not a minimal *rp*-closed set, then $\exists$ (finite) *rp*-closed set V_2 such that $\phi \neq V_2 \subset V_1$. Continuing this process, we have a sequence of *rp*-closed sets $V \supset V_1 \supset V_2 \supset V_3 \supset \ldots\ldots \supset V_k \supset \ldots\ldots$ Since V is a finite set, this process repeats only finitely. Then finally we get a minimal *rp*-closed set $U = V_n$ for some positive integer n.

Corollary 3.1: Let X be a locally finite space and V be a nonempty *rp*-closed set. Then $\exists$ at least one (finite) minimal *rp*-closed set U such that $U \subset V$.
Proof: Let X be a locally finite space and V be a nonempty *rp*-closed set. Let x in V. Since X is locally finite space, we have a finite open set V_x such that x in V_x. Then $V \cap V_x$ is a finite *rp*-closed set. By Theorem 3.6 $\exists$ at least one (finite) minimal *rp*-closed set U such that $U \subset V \cap V_x$. That is $U \subset V \cap V_x \subset V$. Hence $\exists$ at least one (finite) minimal *rp*-closed set U such that $U \subset V$.

Corollary 3.2: If V is finite minimal open set. Then $\exists$ at least one (finite) minimal *rp*-closed set U s.t. $U \subset V$.
Proof: Let V be a finite minimal open set. Then V is a nonempty finite *rp*-closed set. By Theorem 3.6, $\exists$ at least one (finite) minimal *rp*-closed set U such that $U \subset V$.

Theorem 3.7: Let U; U_λ be minimal *rp*-closed sets for any $\lambda \in \Gamma$. If $U \subset \cup_{\lambda \in \Gamma} U_\lambda$, then $\exists\ \lambda \in \Gamma$ s.t. $U = U_\lambda$.
Proof: Let $U \subset \cup_{\lambda \in \Gamma} U_\lambda$. Then $U \cap (\cup_{\lambda \in \Gamma} U_\lambda) = U$. That is $\cup_{\lambda \in \Gamma}(U \cap U_\lambda) = U$. Also by lemma[3.1] (ii), $U \cap U_\lambda = \phi$ or $U = U_\lambda$ for any $\lambda \in \Gamma$. It follows that $\exists$ an element $\lambda \in \Gamma$ such that $U = U_\lambda$.

Theorem 3.8: Let U; U_λ be minimal *rp*-closed sets for any $\lambda \in \Gamma$. If $U = U_\lambda$ for any $\lambda \in \Gamma$, then $(\cup_{\lambda \in \Gamma} U_\lambda) \cap U = \phi$.
Proof: Suppose that $(\cup_{\lambda \in \Gamma} U_\lambda) \cap U \neq \phi$. That is $\cup_{\lambda \in \Gamma}(U_\lambda \cap U) \neq \phi$. Then $\exists$ an element $\lambda \in \Gamma$ such that $U \cap U_\lambda \neq \phi$. By lemma 3.1(ii), we have $U = U_\lambda$, which contradicts the fact that $U \neq U_\lambda$ for any $\lambda \in \Gamma$. Hence $(\cup_{\lambda \in \Gamma} U_\lambda) \cap U = \phi$.
We now introduce maximal *rp*-closed sets in topological spaces as follows.

Definition 3.2: A proper nonempty *rp*-closed $F \subset X$ is said to be maximal *rp*-closed set if any *rp*-closed set containing F is either X or F.

Remark 2: Maximal closed set and maximal *rp*-closed set are not same.

Example 2: In Example 1, {b} is Maximal closed but not Maximal *rp*-closed, {a, b} and {b, c} are Maximal *rp*-closed but not Maximal closed.

Remark 3: From the known results and by the above example we have the following implications:

Theorem 3.9: A proper nonempty subset F of X is maximal *rp*-closed set iff X-F is a minimal *rp*-open set.

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Proof: Let F be a maximal *rp*-closed set. Suppose X-F is not a minimal *rp*-open set. Then ∃ *rp*-open set U ≠ X-F such that $\phi \neq U \subset X\text{-}F$. That is $F \subset X\text{-}U$ and X-U is a *rp*-closed set which is a contradiction for F is a minimal *rp*-open set. Conversely let X-F be a minimal *rp*-open set. Suppose F is not a maximal *rp*-closed set. Then ∃ *rp*-closed set E ≠ F such that $F \subset E \neq X$. That is $\phi \neq X\text{-}E \subset X\text{-}F$ and X-E is a *rp*-open set which is a contradiction for X-F is a minimal *rp*-open set. Therefore F is a maximal *rp*-closed set.

Theorem 3.10: (i) Let F be a maximal *rp*-closed set and W be a *rp*-closed set. Then $F \cup W = X$ or $W \subset F$.
(ii) Let F and S be maximal *rp*-closed sets. Then $F \cup S = X$ or $F = S$.
Proof: (i) Let F be a maximal *rp*-closed set and W be a *rp*-closed set. If $F \cup W = X$, then there is nothing to prove. Suppose $F \cup W \neq X$. Then $F \subset F \cup W$. Therefore $F \cup W = F \Rightarrow W \subset F$.
(ii) Let F and S be maximal *rp*-closed sets. If $F \cup S \neq X$, then we have $F \subset S$ and $S \subset F$ by (i). Therefore F = S.

Theorem 3.11: Let F be a maximal *rp*-closed set. If x is an element of F, then for any *rp*-closed set S containing x, $F \cup S = X$ or $S \subset F$.
Proof: Let F be a maximal *rp*-closed set and x is an element of F. Suppose ∃ *rp*-closed set S containing x such that $F \cup S \neq X$. Then $F \subset F \cup S$ and $F \cup S$ is a *rp*-closed set, as the finite union of *rp*-closed sets is a *rp*-closed set. Since F is a *rp*-closed set, we have $F \cup S = F$. Therefore $S \subset F$.

Theorem 3.12: Let F_α, F_β, F_δ be maximal *rp*-closed sets such that $F_\alpha \neq F_\beta$. If $F_\alpha \cap F_\beta \subset F_\delta$, then either $F_\alpha = F_\delta$ or $F_\beta = F_\delta$
Proof: Given that $F_\alpha \cap F_\beta \subset F_\delta$. If $F_\alpha = F_\delta$ then there is nothing to prove.
If $F_\alpha \neq F_\delta$ then we have to prove $F_\beta = F_\delta$. Now $F_\beta \cap F_\delta = F_\beta \cap (F_\delta \cap X) = F_\beta \cap (F_\delta \cap (F_\alpha \cup F_\beta)$(by thm. 3.10 (ii)) $= F_\beta \cap ((F_\delta \cap F_\alpha) \cup (F_\delta \cap F_\beta)) = (F_\beta \cap F_\delta \cap F_\alpha) \cup (F_\beta \cap F_\delta \cap F_\beta) = (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta)$ (by $F_\alpha \cap F_\beta \subset F_\delta$) $= (F_\alpha \cup F_\delta) \cap F_\beta = X \cap F_\beta$ (Since F_α and F_δ are maximal *rp*-closed sets by theorem[3.10](ii), $F_\alpha \cup F_\delta = X) = F_\beta$. That is $F_\beta \cap F_\delta = F_\beta \Rightarrow F_\beta \subset F_\delta$ Since F_β and F_δ are maximal *rp*-closed sets, we have $F_\beta = F_\delta$ Therefore $F_\beta = F_\delta$

Theorem 3.13: Let F_α, F_β and F_δ be different maximal *rp*-closed sets to each other. Then $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.
Proof: Let $(F_\alpha \cap F_\beta) \subset (F_\alpha \cap F_\delta) \Rightarrow (F_\alpha \cap F_\beta) \cup (F_\delta \cap F_\beta) \subset (F_\alpha \cap F_\delta) \cup (F_\delta \cap F_\beta) \Rightarrow (F_\alpha \cup F_\delta) \cap F_\beta \subset F_\delta \cap (F_\alpha \cup F_\beta)$. Since by theorem 3.10(ii), $F_\alpha \cup F_\delta = X$ and $F_\alpha \cup F_\beta = X \Rightarrow X \cap F_\beta \subset F_\delta \cap X \Rightarrow F_\beta \subset F_\delta$ From the definition of maximal *rp*-closed set it follows that $F_\beta = F_\delta$, which is a contradiction to the fact that F_α, F_β and F_δ are different to each other. Therefore $(F_\alpha \cap F_\beta) \not\subset (F_\alpha \cap F_\delta)$.

Theorem 3.14: Let F be a maximal *rp*-closed set and x be an element of F. Then $F = \cup$ { S: S is a *rp*-closed set containing x such that $F \cup S \neq X$}.
Proof: By theorem 3.12 and fact that F is a *rp*-closed set containing x, we have $F \subset \cup${ S: S is a *rp*-closed set containing x such that $F \cup S \neq X$} – F. Therefore we have the result.

Theorem 3.15: Let F be a proper nonempty cofinite *rp*-closed set. Then ∃ (cofinite) maximal *rp*-closed set E such that $F \subset E$.
Proof: If F is maximal *rp*-closed set, we may set E = F. If F is not a maximal *rp*-closed set, then ∃ (cofinite) *rp*-closed set F_1 such that $F \subset F_1 \neq X$. If F_1 is a maximal *rp*-closed set, we may set $E = F_1$. If F_1 is not a maximal *rp*-closed set, then ∃ a (cofinite) *rp*-closed set F_2 such that $F \subset F_1 \subset F_2 \neq X$. Continuing this process, we have a sequence of *rp*-closed, $F \subset F_1 \subset F_2 \subset \ldots \subset F_k \subset \ldots$. Since F is a cofinite set, this process repeats only finitely. Then, finally we get a maximal *rp*-closed set $E = E_n$ for some positive integer n.

Theorem 3.16: Let F be a maximal *rp*-closed set. If x is an element of X-F. Then $X\text{-}F \subset E$ for any *rp*-closed set E containing x.
Proof: Let F be a maximal *rp*-closed set and x in X-F. $E \not\subset F$ for any *rp*-closed set E containing x. Then $E \cup F = X$ by theorem 3.10(ii). Therefore $X\text{-}F \subset E$.

4. CONCLUSION

In this paper we introduced the concept of minimal rp-closed and maximal rp-closed sets, studied their basic properties.

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