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Somewhat rg-continuous and somewhat rg-open functions

Balasubramanian S[1], Sandhya C[2], Saikumar MDS[3]

1. Department of Mathematics, Govt. Arts College (A), Karur – 639 005, Tamilnadu, India, E-mail: mani55682@rediffmail.com
2. Department of Mathematics, C.S.R. Sarma College, Ongole – 523 001, Andhraparadesh, India, E-mail: sandhya_karavadi@yahoo.co.uk
3. Department of Mathematics, Krishnaveni Degree College, Narasaraopet – 522 601, Andhraparadesh, India, E-mail: mds_sai@yahoo.co.in

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ABSTRACT

In this paper we discuss new type of continuous functions called slightly *rg*−continuous; somewhat *rg*-continuous and somewhat *rg*-open functions; its properties and interrelation with other such functions are studied.

Keywords: slightly continuous functions; slightly semi-continuous functions; slightly pre-continuous; slightly β−continuous functions; slightly γ−continuous functions and slightly ν−continuous functions; somewhat continuous functions; somewhat semi-continuous functions; somewhat pre-continuous; somewhat β−continuous functions; somewhat γ−continuous functions and somewhat ν−continuous functions; somewhat open functions; somewhat semi-open functions; somewhat pre-open; somewhat β−open functions; somewhat γ−open functions and somewhat ν−open functions

AMS-classification Numbers: 54C10; 54C08; 54C05

1. INTRODUCTION

b-open sets are introduced by Andrijevic in 1996. K.R.Gentry introduced somewhat continuous functions in the year 1971. V.K.Sharma and the present authors of this paper defined and studied basic properties of *v*-open sets and *v*-continuous functions in the year 2006 and 2010 respectively. T.Noiri and N.Rajesh introduced somewhat b-continuous functions in the year 2011. Inspired with these developments we introduce in this paper slightly *rg*−continuous, somewhat *rg*-continuous functions and somewhat *rg*-open functions and study its basic properties and interrelation with other type of such functions. Throughout the paper (X, τ) and (Y, σ) (or simply X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned.

2. PRELIMINARIES

2.1. Definition 2.1

$A\subset X$ is called (i) g-closed[rg-closed] if cl $A\subseteq U$ whenever $A\subseteq U$ and U is open in X. (ii)b-open if $A\subset(cl\{A\})^o\cap cl\{A^o\}$.

2.2. Definition 2.2

A function f**:**$X\rightarrow Y$ is said to be

(i) continuous[resp**:** nearly-continuous; rα-continuous; *v*-continuous; α-continuous; semi-continuous; β−continuous; pre-continuous] if inverse image of each open set is open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β -open; preopen].

(ii) nearly-irresolute[resp**:** rα-irresolute; *v*-irresolute; α-irresolute; irresolute; β−irresolute; pre-irresolute] if inverse image of each regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen] set is regular-open[resp**:** rα-open; *v*-open; α-open; semi-open; β-open; preopen].

(iii) almost continuous[resp**:** almost nearly-continuous; almost rα-continuous; almost *v*-continuous; almost α-continuous; almost semi-continuous; almost β−continuous; almost pre-continuous] if for each x in X and each open set (V, f(x)), ∃ an open[resp**:** regular-open; rα-open; *v*-open; α-open; semi-open; β-open; preopen] set (U, x) such that $f(U)\subset(cl(V))^o$.

(iv) somewhat continuous[resp: somewhat b-continuous; somewhat *v*-continuous] if for $U\in\sigma$ and $f^{-1}(U)\neq\varphi$, there exists a non empty open[resp: non empty b-open; non empty *v*-open] set V in X such that $V\neq\varphi$ and $V\subset f^{-1}(U)$.

(v) somewhat-open[resp: somewhat b--open; somewhat *v*-open] provided that if $U\in\tau$ and $U\neq\varphi$, then there exists a non empty b-open set[resp: non empty b-open; non empty *v*-open] V in Y such that $V\neq\varphi$ and $V\subset f(U)$.

(vi) somewhat *v*-irresolute if for $U\in vO(\sigma)$ and $f^{-1}(U)\neq\varphi$, there exists a non-empty *v*-open set V in X such that $V\subset f^{-1}(U)$.

2.3. Lemma 2.1

(i) Let A and B be subsets of a space X, if $A\in RGO(X)$ and $B\in RO(X)$, then $A\cap B\in RGO(B)$. (ii)Let $A\subset B\subset X$, if $A\in RGO(B)$ and $B\in RO(X)$, then $A\in RGO(X)$.

2.4. Definition 2.5

X is said to be resolvable[resp: b-resolvable] if both A and X - A are dense[resp: b-dense] in X. Otherwise, X is called irresolvable[resp: b-irresolvable].

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3. SOMEWHAT rg-CONTINUOUS FUNCTION

3.1. Definition 3.1

A function f is said to be somewhat rg-continuous if for $U\in\sigma$ and $f^{-1}(U) \neq \varphi$, there exists a non-empty rg-open set V in X such that $V\subset f^{-1}(U)$.

It is clear that every continuous function is somewhat continuous and every somewhat continuous is somewhat rg-continuous. But the converses are not true by Example 1 of [21] and the following example.

Example 3.1: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{b\}, \{a, b\}, \{b, c\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, X\}$. The function $f:(X, \tau)\to (X, \sigma)$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is somewhat rg-continuous.

Example 3.2: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{b, c\}, X\}$, $\sigma = \{\varphi, \{b\}, \{a, c\}, X\}$ and $\eta = \{\varphi, \{a\}, X\}$. Then the identity functions $f:(X, \tau)\to (X, \sigma)$ and $g:(X, \sigma)\to (X; \eta)$ and $g\bullet f$ are somewhat rg-continuous.

However, we have the following

3.2. Theorem 3.1

If f is somewhat rg-continuous and g is continuous, then $g\bullet f$ is somewhat rg-continuous.

3.3. Corollary 3.1

If f is somewhat rg-continuous and g is r-continuous[resp: r-irresolute], then $g\bullet f$ is somewhat rg-continuous.

3.4. Theorem 3.2

For a surjective function f, the following statements are equivalent: (i) f is somewhat rg-continuous. (ii) If C is a closed subset of Y such that $f^{-1}(C) \neq X$, then there is a proper rg-closed subset D of X such that $f^{-1}(C) \subset D$. (iii) If M is a rg-dense subset of X, then $f(M)$ is a dense subset of Y.

Proof: (i) ⇒(ii): Let C be a closed subset of Y such that $f^{-1}(C) \neq X$. Then Y-C is an open set in Y such that $f^{-1}(Y\text{-}C) = X - f^{-1}(C) \neq \varphi$ By (i), there exists a rg- open set $V\in RGO(X)$ such that $V \neq \varphi$ and $V\subset f^{-1}(Y\text{-}C) = X - f^{-1}(C)$. This means that $X\text{-}V\supset f^{-1}(C)$ and $X - V = D$ is a proper rg-closed set in X.

(ii) ⇒(i): Let $U\in\sigma$ and $f^{-1}(U) \neq \varphi$ Then Y-U is closed and $f^{-1}(Y\text{-}U) = X\text{-}f^{-1}(U) \neq X$. By (ii), there exists a proper rg-closed set D such that $D\supset f^{-1}(Y\text{-}U)$. This implies that $X\text{-}D \subset f^{-1}(U)$ and X-D is rg-open and $X\text{-}D \neq \varphi$.

(ii) ⇒(iii): Let M be a rg-dense set in X. Suppose that $f(M)$ is not dense in Y. Then there exists a proper closed set C in Y such that $f(M) \subset C\subset Y$. Clearly $f^{-1}(C) \neq X$. By (ii), there exists a proper rg-closed set D such that $M\subset f^{-1}(C) \subset D\subset X$. This is a contradiction to the fact that M is rg-dense in X.

(iii) ⇒(ii): Suppose (ii) is not true. there exists a closed set C in Y such that $f^{-1}(C) \neq X$ but there is no proper rg-closed set D in X such that $f^{-1}(C) \subset D$. This means that $f^{-1}(C)$ is rg-dense in X. But by (iii), $f(f^{-1}(C)) = C$ must be dense in Y, which is a contradiction to the choice of C.

3.5. Theorem 3.3

Let f be a function and $X = A\cup B$, where $A,B\in\tau(X)$. If the restriction functions $f_{/A}: (A; \tau_{/A}) \to (Y, \sigma)$ and $f_{/B}:(B; \tau_{/B}) \to (Y, \sigma)$ are somewhat rg-continuous, then f is somewhat rg-continuous.

Proof: Let $U\in\sigma$ such that $f^{-1}(U) \neq \varphi$. Then $(f_{/A})^{-1}(U) \neq \varphi$ or $(f_{/B})^{-1}(U) \neq \varphi$ or both $(f_{/A})^{-1}(U) \neq \varphi$ and $(f_{/B})^{-1}(U) \neq \varphi$. Suppose $(f_{/A})^{-1}(U) \neq \varphi$, Since $f_{/A}$ is somewhat rg-continuous, there exists a rg-open set V in A such that $V\neq \varphi$ and $V\subset (f_{/A})^{-1}(U) \subset f^{-1}(U)$. Since V is rg-open in A and A is r-open in X, V is rg-open in X. Thus f is somewhat rg-continuous. The proof of other cases are similar.

3.6. Definition 3.2

If X is a set and τ and σ are topologies on X, then τ is said to be equivalent[resp: rg- equivalent] to σ provided if $U\in\tau$ and $U \neq \varphi$, then there is an open[resp:rg-open] set V in *X* such that $V \neq \varphi$ and $V \subset U$ and if $U\in\sigma$ and $U \neq \varphi$, then there is an open[resp:rg-open] set V in (X, τ) such that $V\neq \varphi$ and $U \supset V$.

3.7. Definition 3.3

$A\subset X$ is said to be rg-dense in X if there is no proper rg-closed set C in X such that $M\subset C\subset X$.

Now, consider the identity function f and assume that τ and σ are rg-equivalent. Then f and f^{-1} are somewhat rg-continuous. Conversely, if the identity function f is somewhat rg-continuous in both directions, then τ and σ are rg-equivalent.

3.8. Theorem 3.4

Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat rg-continuous surjection and τ^* be a topology for X, which is rg-equivalent to τ. Then $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat rg-continuous.

Proof: Let $V\in\sigma$ such that $f^{-1}(V) \neq \varphi$. Since f is somewhat rg-continuous, there exists a nonempty rg-open set U in (X, τ) such that $U\subset f^{-1}(V)$. But by hypothesis τ^* is rg-equivalent to τ. Therefore, there exists a rg-open set $U^*\in (X; \tau^*)$ such that $U^*\subset U$. But $U\subset f^{-1}(V)$. Then $U^*\subset f^{-1}(V)$; hence $f:(X, \tau^*)\to (Y, \sigma)$ is somewhat rg-continuous.

3.9. Theorem 3.5

Let $f:(X, \tau)\to (Y, \sigma)$ be a somewhat rg-continuous surjection and σ^* be a topology for Y, which is equivalent to σ. Then $f:(X, \tau)\to (Y, \sigma^*)$ is somewhat rg-continuous.

Proof: Let $V^*\in\sigma^*$ such that $f^{-1}(V^*) \neq \varphi$. Since σ^* is equivalent to σ, there exists a nonempty open set V in (Y, σ) such that $V\subset V^*$. Now $\varphi = f^{-1}(V) \subset f^{-1}(V^*)$. Since f is somewhat rg-continuous, there exists a nonempty rg-open set U in (X, τ) such that $U\subset f^{-1}(V)$. Then $U\subset f^{-1}(V^*)$; hence $f:(X, \tau)\to (Y, \sigma^*)$ is somewhat rg-continuous.

4. SOMEWHAT rg-OPEN FUNCTION

4.1. Definition 4.1

A function f is said to be somewhat rg-open provided that if $U\in\tau$ and $U\neq \varphi$, then there exists a non-empty rg-open set V in Y such that $V\subset f(U)$.

Example 4.1: Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, X\}$ and $\sigma = \{\varphi, \{a\}, \{b, c\}, X\}$. The function $f: (X, \tau)\to (X, \sigma)$ defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$ is somewhat rg-open, somewhat rg-open and somewhat open.

4.2. Theorem 4.1

Let f be an r-open function and g somewhat rg-open. Then $g\bullet f$ is somewhat rg-open.

4.3. Theorem 4.2

For a bijective function f, the following are equivalent: (i) f is somewhat rg-open. (ii) If C is a closed subset of X, such that $f(C) \neq Y$, then there is a rg-closed subset D of Y such that $D\neq Y$ and $D\supset f(C)$.

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Proof: (i) $\Rightarrow$(ii): Let C be any closed subset of X such that f(C) $\neq$ Y. Then X-C is open in X and X-C $\neq \varphi$. Since f is somewhat rg-open, there exists a rg-open set V$\neq \varphi$ in Y such that V$\subset f$(X-C). Put D = Y-V. Clearly D is rg-closed in Y and we claim D$\neq$ Y. If D = Y, then V = φ, which is a contradiction. Since V$\subset f$(X-C), D = Y-V $\supset$ (Y- f(X-C)) = f(C).
(ii) $\Rightarrow$(i): Let U be any nonempty open subset of X. Then C = X-U is a closed set in X and f(X-U) = f(C) = Y- f(U) implies f(C) $\neq$ Y. Therefore, by (ii), there is a rg-closed set D of Y such that D$\neq$ Y and f(C) $\subset$ D. Clearly V = Y-D is a rg-open set and V$\neq \varphi$. Also, V = Y-D $\subset$ Y- f(C) = Y- f(X-U) = f(U).

4.4. Theorem 4.3

The following statements are equivalent: (i) f is somewhat rg-open. (ii) If A is a rg-dense subset of Y, then f^{-1}(A) is a dense subset of X.
Proof: (i) $\Rightarrow$(ii): Suppose A is a rg-dense set in Y. If f^{-1}(A) is not dense in X, then there exists a closed set B in X such that f^{-1}(A) $\subset$ B$\subset$ X. Since f is somewhat rg-open and X-B is open, there exists a nonempty rg-open set C in Y such that C$\subset f$(X-B). Therefore, C$\subset f$(X-B) $\subset f(f^{-1}$(Y-A)) $\subset$ Y-A. That is, A$\subset$ Y-C $\subset$ Y. Now, Y-C is a rg-closed set and A$\subset$ Y-C $\subset$ Y. This implies that A is not a rg-dense set in Y, which is a contradiction. Therefore, f^{-1}(A) is a dense set in X.
(ii) $\Rightarrow$(i): Suppose A is a nonempty open subset of X. We want to show that rg(f(A))$^{o}\neq \varphi$. Suppose rg(f(A))^{o} = φ. Then, rgcl{(f(A))} = Y. Therefore, by (ii), f^{-1}(Y - f(A)) is dense in X. But f^{-1}(Y- f(A)) $\subset$ X-A. Now, X-A is closed. Therefore, f^{-1}(Y- f(A)) $\subset$ X-A gives X = cl{(f^{-1}(Y- f(A)))} $\subset$ X-A. This implies that A = φ, which is contrary to A$\neq \varphi$. Therefore, rg(f(A))$^{o}\neq \varphi$. Hence f is somewhat rg-open.

4.5. Theorem 4.4

Let f be somewhat rg-open and A be any r-open subset of X. Then $f_{/A}$:(A; $\tau_{/A}$) $\rightarrow$ (Y, σ) is somewhat rg-open.
Proof: Let U$\in\tau_{/A}$ such that U $\neq \varphi$. Since U is r-open in A and A is r-open in X, U is r-open in X and since by hypothesis f is somewhat rg-open function, there exists a rg-open set V in Y, such that V$\subset f$(U). Thus, for any open set U of A with U$\neq \varphi$, there exists a rg-open set V in Y such that V$\subset f$(U) which implies $f_{/A}$ is a somewhat rg-open function.

4.6. Theorem 4.5

Let f be a function and X = A$\cup$ B, where A,B$\in\tau$(X). If the restriction functions $f_{/A}$ and $f_{/B}$ are somewhat rg-open, then f is somewhat rg-open.
Proof: Let U be any open subset of X such that U $\neq \varphi$. Since X = A$\cup$ B, either A$\cap$ U $\neq \varphi$ or B$\cap$U $\neq \varphi$ or both A$\cap$U $\neq \varphi$ and B$\cap$U $\neq \varphi$. Since U is open in X, U is open in both A and B.
Case (i): If A$\cap$U $\neq \varphi$ open in A. Since $f_{/A}$ is somewhat rg-open, there exists a rg-open set V of Y such that V$\subset f$(U$\cap$A) $\subset f$(U), which implies that f is a somewhat rg-open.
Case (ii): If B$\cap$U $\neq \varphi$ open in B. Since $f_{/B}$ is somewhat rg-open, there exists a rg-open set V in Y such that V$\subset f$(U$\cap$B) $\subset f$(U), which implies that f is somewhat rg-open.
Case (iii): If both A$\cap$U $\neq \varphi$ and B$\cap$U $\neq \varphi$. Then by case (i) and (ii) f is somewhat rg-open.
Remark 3: Two topologies τ and σ for X are said to be rg-equivalent if and only if the identity function f: (X, τ)$\rightarrow$ (Y, σ) is somewhat rg-open in both directions.

4.7. Theorem 4.6

Let f: (X, τ)$\rightarrow$ (Y, σ) be a somewhat almost open function. Let τ* and σ* be topologies for X and Y, respectively such that τ* is equivalent to τ and σ* is rg-equivalent to σ. Then f: (X; τ*)$\rightarrow$ (Y; σ*) is somewhat rg-open.

5. COVERING AND SEPARATION PROPERTIES OF SWT.*rg*.c. FUNCTIONS

5.1. Theorem 5.1

If f is swt.rg.c.[resp**:** swt.rg.c] surjection and X is *rg*-compact, then Y is compact.
Proof: Let {G_i**:**i$\in$ I} be any open cover for Y. Then each G_i is open in Y and hence each G_i is clopen in Y. Since f is sl.rg.c., $f^{-1}(G_i)$ is *rg*-open in X. Thus {$f^{-1}(G_i)$} forms a *rg*-open cover for X and hence have a finite subcover, since X is *rg*-compact. Since f is surjection, Y = f(X) = $\cup^{n}_{i=1}G_i$. Therefore Y is compact.

5.2. Corollary 5.1

If f is swt.c.[resp**:** swt.r.c] surjection and X is *rg*-compact, then Y is compact.

5.3. Theorem 5.2

If f is swt.rg.c., surjection and X is *rg*-compact[*rg*-lindeloff] then Y is mildly compact[mildly lindeloff].
Proof: Let {U_i**:**i$\in$ I} be open cover for Y. For each x in X, $\exists$ $\alpha_x\in$ I such that f(x) $\in U_{\alpha x}$ and $\exists$ $V_x\in$ *RGO*(X, x) such that $f(V_x)\subset U_{\alpha x}$. Since the family {V_i**:**i$\in$ I} is a cover of X by *rg*-open sets of X, $\exists$ a finite subset I_0 of I such that X$\subset$ {V_x**:**x$\in$ I_0}. Therefore Y$\subset$ $\cup$ {$f(V_x)$**:**x$\in$ I_0}$\subset$ $\cup$ {$U_{\alpha x}$**:**x$\in$ I_0}. Hence Y is mildly compact.

5.4. Corollary 5.2

(i) If f is swt.rg.c[resp**:** swt.r.c] surjection and X is *rg*-compact[*rg*-lindeloff] then Y is mildly compact[mildly lindeloff]. (ii) If f is swt.rg.c.[resp**:** swt.rg.c; swt.r.c] surjection and X is locally *rg*-compact{resp**:***rg*-Lindeloff; locally *rg*-lindeloff}, then Y is locally compact{resp**:** Lindeloff; locally lindeloff}.
(iii)If f is swt.rg.c., surjection and X is semi-compact[semi-lindeloff] then Y is mildly compact[mildly lindeloff].
(iv) If f is swt.rg.c[swt.r.c.], surjection and X is locally *rg*-compact{resp**:** *rg*-lindeloff; locally *rg*-lindeloff} then Y is locally mildly compact{resp**:** locally mildly lindeloff}.

5.5. Theorem 5.3

If f is swt.rg.c., surjection and X is s-closed then Y is mildly compact[mildly lindeloff].
Proof: Let {V_i **:** $V_i\in$ σ(Y); i$\in$ I} be a cover of Y, then {$f^{-1}(V_i)$ **:** i$\in$ I} is *rg*-open cover of X[by Thm 3.1] and so there is finite subset I_0 of I, such that {$f^{-1}(V_i)$**:**i$\in$ I_0} covers X. Therefore {V_i **:** i$\in$ I_0} covers Y since f is surjection. Hence Y is mildly compact.

5.6. Corollary 5.3

If f is swt.rg.c[resp**:** swt.r.c.] surjection and X is s-closed then Y is mildly compact[mildly lindeloff].

5.7. Theorem 5.4

If f is swt.rg.c.,[resp**:** swt.c.; swt.r.c.] surjection and X is *rg*-connected, then Y is connected.
Proof: If Y is disconnected, then Y = A$\cup$ B where A and B are disjoint clopen sets in Y. Since f is sl.rg.c. surjection, X = f^{-1}(Y) = f^{-1}(A) $\cup$ f^{-1}(B) where f^{-1}(A) f^{-1}(B) are disjoint *rg*-open sets in X, which is a contradiction for X is *rg*-connected. Hence Y is connected.

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5.8. Corollary 5.4

The inverse image of a disconnected space under a swt.rg.c.,[resp**:** swt.r.c.] surjection is *rg*-disconnected.

5.9. Theorem 5.5

If *f* is swt.rg.c..[resp**:** swt.rg.c.], injection and Y is UT_i, then X is rg_i i = 0, 1, 2.

Proof: Let $x_1 \neq x_2 \in$ X, since *f* is injective and Y is UT_2 $\exists$ $V_j \in CO(Y, f(x_j))$ and $\cap V_j = \phi$ for j = 1,2. By Theorem 3.1, $x_j \in f^{-1}(V_j) \in RGO(X)$ for j = 1,2 and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is rg_2.

5.10. Theorem 5.6

If *f* is swt.rg.c., injection; closed and Y is UT_i, then X is rgg_i i = 3, 4.

Proof:(i) Let x in X and F be disjoint closed subset of X not containing x, then *f*(x) and *f*(F) be disjoint closed subset of Y not containing *f*(x), since *f* is closed and injection. Since Y is ultraregular, *f*(x) and *f*(F) are separated by disjoint clopen sets U and V respectively. Hence $x \in f^{-1}(U)$; $F \subseteq f^{-1}(V)$, $f^{-1}(U)$; $f^{-1}(V) \in RGO(X)$ and $f^{-1}(U) \cap f^{-1}(V) = \phi$. Thus X is rgg_3.

(ii) Let F_j and $f(F_j)$ are disjoint closed subsets of X and Y respectively for j = 1,2, since *f* is closed and injection. For Y is ultranormal, $f(F_j)$ are separated by disjoint clopen sets V_j respectively for j = 1,2. Hence $F_j \subseteq f^{-1}(V_j)$ and $f^{-1}(V_j) \in RGO(X)$ and $\cap f^{-1}(V_j) = \phi$ for j = 1,2. Thus X is rgg_4.

5.11. Theorem 5.7

If *f* is swt.*rg*.c., injection and

(i) Y is UC_i[resp**:** UD_i] then X is rgC_i[resp**:** rgD_i] i = 0, 1, 2. (ii)Y is UR_i, then X is rg-R_i i = 0, 1.

5.12. Theorem 5.8

If *f* is swt.*rg*.c.[resp**:** swt.c; swt.r.c] and Y is UT_2, then the graph G(*f*) of *f* is *rg*-closed in X$\times$Y.

Proof: Let $(x, y) \notin G(f)$ implies $y \neq f(x)$ implies $\exists$ disjoint V; $W \in CO(Y)$ such that $f(x) \in V$ and $y \in W$. Since *f* is sl.rg.c., $\exists$ $U \in RGO(X)$ such that $x \in U$ and $f(U) \subset W$ and $(x, y) \in U \times V \subset X \times Y - G(f)$. Hence G(*f*) is *rg*-closed in X$\times$Y.

5.13. Theorem 5.9

If *f* is swt.rg.c.[resp**:** swt.c; swt.r.c] and Y is UT_2, then $A = \{(x_1, x_2)|\ f(x_1) = f(x_2)\}$ is *rg*-closed in X$\times$X.

Proof: If $(x_1, x_2) \in X \times X - A$, then $f(x_1) \neq f(x_2)$ implies $\exists$ disjoint $V_j \in CO(Y)$ such that $f(x_j) \in V_j$, and since *f* is sl.rg.c., $f^{-1}(V_j) \in RGO(X, x_j)$ for j = 1,2. Thus $(x_1, x_2) \in f^{-1}(V_1) \times f^{-1}(V_2) \in RGO(X \times X)$ and $f^{-1}(V_1) \times f^{-1}(V_2) \subset X \times X - A$. Hence A is *rg*-closed.

5.14. Theorem 5.10

If *f* is swt.r.c.[resp**:** swt.c.]; *g* is swt.*rg*.c[resp**:** swt.c]; and Y is UT_2, then E = {x in X **:** *f*(x) = *g*(x)} is *rg*-closed in X.

6. CONCLUSION

In this paper we defined somewhat *rg*-continuous functions, studied its properties and their interrelations with other types of somewhat-continuous functions.

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