Domination Number on Balanced Signed Graphs

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ABSTRACT

A signed graph based on F is an ordinary graph F with each edge marked as positive or negative. Such a graph is called balanced if each of its cycles includes an even number of negative edges. We find the domination set on the vertices, on bipartite graphs and show that graphs have domination Number on signed graphs, such that a signed graph G may be converted into a balanced graph by changing the signs of d edges. We investigate the number D(F) defined as the largest d(G) such that G is a signed graph based on F. If F is the complete bipartite graph with t vertices in each part, then D(F) ≤ \( \frac{1}{2} t^2 - c t^{3/2} \) for some positive constant c.

Definition
Hoffman Graph: The Hoffman graph is the bipartite graph on 16 nodes and 32 edges illustrated above that is cospectral to the tesseract graph \( Q_4 \) (Hoffman 1963, van Dam and Haemers 2003). \( Q_4 \) and the Hoffman graph are therefore not determined by their spectrum. Its girth, graph diameter, graph spectrum, and characteristic polynomial are the same as those of \( Q_4 \), but its graph radius is 3 compared to the value 4 for \( Q_4 \).

In graph theory, a dominating set for a graph \( G = (V, E) \) is a subset \( D \) of \( V \) such that every vertex not in \( D \) is adjacent to at least one member of \( D \). The domination number \( \gamma(G) \) is the number of vertices in a smallest dominating set for \( G \).
'V' Denotes the vertices, 
'*\phi*' denotes the domination set

\[ D(\phi) \{V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8\} = 4 \]

Figure: domination number on balanced signed bipartite graph

Illustration:
There are 17 possible balanced signed graphs on this hoffman's bipartite graph

According to the definition, \[ D(\phi) \leq \frac{1}{2} t^2 - ct^2 \]

Where 't' is number Of vertices \( t = 16 \)

\[ D(\phi) \leq \frac{1}{2}(16)^2 - c(16)^{3/2} \]

\[ D(\phi) \leq \frac{1}{2}(256) - c(64) \]

\[ D(\phi) \leq 128 - (64)c \]

Thus the condition satisfies

A complete bipartite graph

\( K_{r,r} \) is a circulant graph (Skiena 1990, p. 99), Specifically \( C_{i_1, i_2, \ldots, 2\lfloor \frac{n}{2} \rfloor + 1} \{ n \} \), where \( \lfloor x \rfloor \) is the floor function.
“V” denotes the vertices of the Graph

Denotes the Domination Set

\[ D(f) \text{ of } \{V_1, V_2, \ldots, V_{10}\} = 5 \]

There are 21 possible balanced signed graphs on this bipartite graph \( K_{10,10} \)

According to the definition, \[ D(f) \leq \frac{t}{2} \cdot \frac{c t^3}{2} \]

Where ‘t’ is number of vertices \( t = 40 \)

\[ D(f) \leq 800 - c(252.98) \] :

where \( c \) is some Positive constant

\[ D(f) \leq 800 \] :

where \( c \) is Zero

\[ D(f) \leq 800 - (252.98)c \] :

where \( c \) is some Negative constant

Thus the condition satisfies

The Following Graphs are the Single connected domination number Balanced Signed Graphs:

An \((n,k)\)-banana tree, as defined by Chen et al. (1997), is a graph obtained by connecting one leaf of each of \( n \) copies of an \( k \) Star Graph with a single root vertex that is distinct from all the stars.

The \((n,k)\)-banana tree has Rank Polynomial

\[ R(x) = (1 + x)^{nk} \]
“V” denotes the vertices of the Graph

Denotes the Domination Set

There are 14 possible balanced signed graphs which could be formed from the above $B(5,5)$ graph.

Where ‘t’ is number Of vertices t=26

$$D(f) \leq (26)^2 - \frac{c(26)^{3/2}}{2}$$

$D(f) \leq 338 - (132.57)c$ \quad \begin{cases} \text{where } c \text{ is some Positive constant} \\
\text{where } c \text{ is Zero} \\
\text{where } c \text{ is some Negative constant} \end{cases}$

Thus the condition satisfies

Friendship Graph:
The Friendship Graph is denoted by $F_n$. Where the below figure is $F_8$.
Vertices $2n+1$, and Edges $3n$. 

Where \( t \) is number of vertices \( t = 17 \)

\[ D(f) \] of \( \{v_1, v_2, \ldots, v_{16}\} = 2 \] and \( D(f) \) of \( \{v_17\} = 16 \)

There are 13 possible balanced signed graphs which could be formed from the above graph.

\( v \) denotes the vertices of the graph.

\( D(f) \) denotes the Domination Set.
The Following are the Two connected domination number balanced signed graphs:

Butterfly Networks

The n-dimensional butterfly graph has $2^n (n+1)$ vertices and $2^{n+1} n$ edges. BF (2)

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\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{butterfly_graph.png}
\caption{Butterfly Graph}
\end{figure}
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Thus the condition satisfies

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D(f) \leq \frac{1}{2} (17)^2 - c(17)^{3/2}
D(f) \leq 144.5 - c(70.09) \quad \leftarrow \text{where } c \text{ is some Positive constant}
D(f) \leq 144.5 \quad \leftarrow \text{where } c \text{ is Zero}
D(f) \leq 144.5 - (70.09)c \quad \leftarrow \text{where } c \text{ is some Negative constant}
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Where the condition satisfies

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D(f) \leq \frac{1}{2} (12)^2 - c(12)^{3/2}
D(f) \leq 72 - c(41.56) \quad \leftarrow \text{where } c \text{ is some Positive constant}
D(f) \leq 72 \quad \leftarrow \text{where } c \text{ is Zero}
D(f) \leq 72 - (41.56)c \quad \leftarrow \text{where } c \text{ is some Negative constant}
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Thus the condition satisfies
**Book Graph:**

The $m$-book graph is defined as the graph Cartesian product $S_{m+1} \times P_2$, where $S_m$ is a star graph and $P_2$ is the path graph on two nodes. The generalization of the book graph to $n$ “stacked” pages is the $(m, n)$-stacked book graph.

There are 9 possible balanced signed graphs which could be formed from the above $B_5$ graph. Where 't' is number of vertices $t=12$

\[
D(f) \leq \frac{3}{2} \left(12\right)^{2} - \frac{1}{2} \left(12\right)^{3/2} \\
D(f) \leq 72 - c\left(41.56\right) \quad \text{where } c \text{ is some Positive constant} \\
D(f) \leq 72 - 41.56c \quad \text{where } c \text{ is Zero} \\
D(f) \leq 72 - 41.56c \quad \text{where } c \text{ is some Negative constant} \\
Thus the condition satisfies
Conclusion

We have proved that some splitting graphs of Standard graph are with Domination Number on balanced signed graph conditions. We found out that, Banana Tree and Friendship graphs has single Connected domination number on Balanced signed graphs, and Butterfly Network, Book Graph has two connected domination number on Balanced signed graphs. And hence satisfied the conditions of Balanced Signed Bipartite Graph.

REFERENCES

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