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Time behavior of superconducting resonators in quantum circuits for quantum information processing

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ABSTRACT

The flux-driven amplification of a superconducting resonator is investigated using the adiabatic invariant method. An adiabatic invariant is derived from the Hamiltonian dynamics and the energy expression, under the assumption that the parameters of the system vary adiabatically. By utilizing the characteristics of the adiabatic invariant, the equation for the time behavior of the energy of the resonator is obtained and it is illustrated for some particular cases. There has been a nearly periodic change of the energy during parametrical amplification of the readout signal. We confirmed that the fluctuation of the energy becomes large as the pumping strength of the resonator increases. In case that the frequency of the external force deviates from the resonance frequency, the pattern of the energy variation becomes somewhat irregular.

Keywords: Superconducting Circuit, Resonator, Adiabatic Invariant, Energy

Abbreviations: SQUID – superconducting quantum interference device

1. INTRODUCTION

Recent remarkable progress for the fabrication of superconducting circuits in nanomechanical quantum domain has opened up a new possibility for the realization of quantum information science via diverse implementations concerning the techniques of storing, processing, and amplifying binary digital quantum signals (Schoelkopf and Girvin, 2008). The use of superconducting circuits in quantum signal processing, such as non-demolition readout of qubit signals (Lupascu et al., 2007), generation of a pair of entangled microwave signals (Marquardt, 2007), and quantum feedback control (Ahn et al., 2002), may lead to the accomplishment of superconducting quantum computing through the scaling of a large numbers of qubits. Non-demolition high fidelity readout of qubit states has been demonstrated from diverse experiments using superconducting circuits in a similar way to that of the cavity quantum electrodynamics (Lupascu et al., 2007).

Though superconducting circuits of the flux qubit play a key role in quantum information processing, the readout of the qubit state signals, in order for the process and control of computing algorithms, requires a high-fidelity resonator (Johnson, 2012). Wide-band amplifiers without noise are indispensable for executing a dispersive readout of the state of superconducting qubits with desired amplitude. We consider an underdamped superconducting resonator which is parametrically driven by a field. If the frequency of the driving field is twice the resonator eigenfrequency, the system is degenerated parametric resonator, whereas otherwise the system is nondegenerated parametric one. An appropriate modulation of parametric resonator leads to the amplification of the readout resonator signals in time.

The time behavior of a parametrically driven flux qubit resonator will be investigated in this work. For this purpose, the Hamiltonian method and the adiabatic invariant method for the superconducting electric circuits of the resonator will be used. From the equation for magnetic flux inside the resonator, we will construct a time-dependent Hamiltonian of the system. From the expression of the energy of the resonator, an adiabatic invariant will be evaluated rigorously. By means of the characteristics of the adiabatic invariant, the time evolution of the energy will be illustrated and analyzed.

2. HAMILTONIAN DESCRIPTION

Amplification of a signal is necessary for high-fidelity readout of information signals from a quantum device in quantum information processing systems. An amplifier enhances the magnitude of weak signals of qubit states to an enough level of the amplitude in order that they can be processed in information processing devices. If the amplification strength (pump strength) is sufficiently small in a superconducting resonator, the dynamics of the system is similar to that of the Duffing oscillator which exhibits bifurcation and bistability of the response (Nayfeh and Mook, 1979).

The fabrication of a superconducting qubit quantum system requires a superconducting quantum interference device (SQUID) which is a micrometer-size superconducting loop interrupted by two or more non-metal junctions which are called Josephson junctions. Superconducting circuits of SQUID exhibit the Josephson effect which is the flow of supercurrent through a Josephson junction without any voltage supply. Such current induces a magnetic flux ϕ in the superconducting ring. Flux qubit resonators utilize magnetic flux as the indication tool of qubit states. Let us consider a resonator in which the magnetic flux follows an extended Duffing equation of the form (Krantz, 2013)

$$\frac{d^2\phi}{dt^2}+2\Gamma\frac{d\phi}{dt}+\left[\omega_r^2+\varepsilon\cos(\omega_p t)-\frac{\beta\varepsilon^2}{4\Gamma\omega_p}[1-\cos(2\omega_p t)]\right]\phi-\alpha\left[1-\frac{3\lambda}{4\Gamma\omega_p}\varepsilon\cos(\omega_p t)\right]\phi^3=\xi(t), \quad (1)$$

where ω_r is the resonant frequency, ω_p is the pumping frequency, α is the Duffing nonlinear term, ε is the strength of pumping, β is a dimensionless parameter, $\Gamma=\omega_r/(2Q)$, Q is the quality factor, $\xi(t)$ is an external force, λ is a correction to the Duffing nonlinearity caused by a modulation of α through the pumping. It is noticeable that the nonlinear term that involves α is produced by the superconducting quantum interference.

A similar expression of Eq. (1) is also represented in Eq. (2.37) of Ref. [Krantz, 2013] in terms of an other variable φ instead of ϕ, where $\varphi=2\pi\phi/\phi_0$, ϕ_0 is the magnetic flux quantum that is given by $\phi_0=\pi\hbar/e$, $\hbar$ is the Planck constant divided by 2π, and e is the elementary charge. Equation (1) can be easily converted to that of Ref. [Krantz, 2013] by just replacing ϕ with φ and by executing a rescaling of α. However, for the simplification of various mathematical expressions that will be appeared in the further development of the theory, our expression, Eq. (1), is more preferable in this case than the one with φ (Russer and Russer, 2012).

We apply Hamiltonian formulations of dynamical systems that provide powerful methodologies with a keen insight for analyzing the characteristics of the system. The mathematical treatment of Eq. (1) as a whole may be not an easy task, because it is a complicated time function and involves a nonlinear term. Let us neglect the nonlinear term for simplicity from now on ($\alpha \to 0$). Then, the Hamiltonian of the system is given by

$$H = e^{-2\Gamma t}\frac{q^2}{2C} + \frac{1}{2}e^{2\Gamma t}C[\Omega^2(t)\phi^2 - 2\xi(t)\phi], \tag{2}$$

where C is the capacitance of the resonator, q is the charge stored in C, and

$$\Omega(t) = \left[\omega_r^2 + \varepsilon\cos(\omega_p t) - \frac{\beta\varepsilon^2}{4\Gamma\omega_p}[1-\cos(2\omega_p t)]\right]^{1/2}. \tag{3}$$

If we follow Russer and Russer's opinion (Russer and Russer, 2012), the magnetic flux ϕ and the charge q play the same roles as position and momentum in mechanical systems, respectively. On the other hand, other opinion (Choi et al., 2013) is that ϕ and q correspond to the momentum and position in mechanical systems, respectively. Both opinions may be acceptable according to the points of view for specific treatments of the system.

Using the Hamilton equations (Russer and Russer, 2012)

$$\frac{dq}{dt} = \frac{\partial H}{\partial \phi}, \qquad \frac{d\phi}{dt} = -\frac{\partial H}{\partial q}, \tag{4}$$

we can easily prove through a proper algebra that the Hamiltonian, Eq. (2), yields Eq. (1) with $\alpha = 0$. The system described by Eq. (2) is a kind of dissipative system because Γ is a dissipation coefficient. In general, for a dissipative system, the energy is different from the Hamiltonian. In this case, the energy is represented as (Yeon et al., 1987)

$$E(t) = e^{-4\Gamma t}\frac{q^2}{2C} + \frac{1}{2}C[\Omega^2(t)\phi^2 - 2\xi(t)\phi]. \tag{5}$$

In the next section, we will investigate the time evolution of $E(t)$.

3. ANALYSIS UNDER ADIABATIC LIMIT

To investigate the time behavior of the resonator system, we use the adiabatic invariant method under the suppositions that the parameters of the system varies slowly and the pumping strength ε is sufficiently small. For this purpose, we can reexpress Eq. (5) as

$$\frac{q^2}{A^2(t)} + \frac{[\phi - \xi(t)/\Omega^2(t)]^2}{B^2(t)} = 1, \tag{6}$$

where

$$A(t) = \left[2Ce^{4\Gamma t}\left(E + \frac{C\xi^2(t)}{2\Omega^2(t)}\right)\right]^{1/2}, \tag{7}$$

$$B(t) = \left[\frac{2}{C\Omega^2(t)}\left(E + \frac{C\xi^2(t)}{2\Omega^2(t)}\right)\right]^{1/2}. \tag{8}$$

According to the fundamental mechanics, the adiabatic invariant of the system is obtained from

$$J = \oint q\, d\phi. \tag{9}$$

Using Eq. (6) with Eqs. (7) and (8), we have

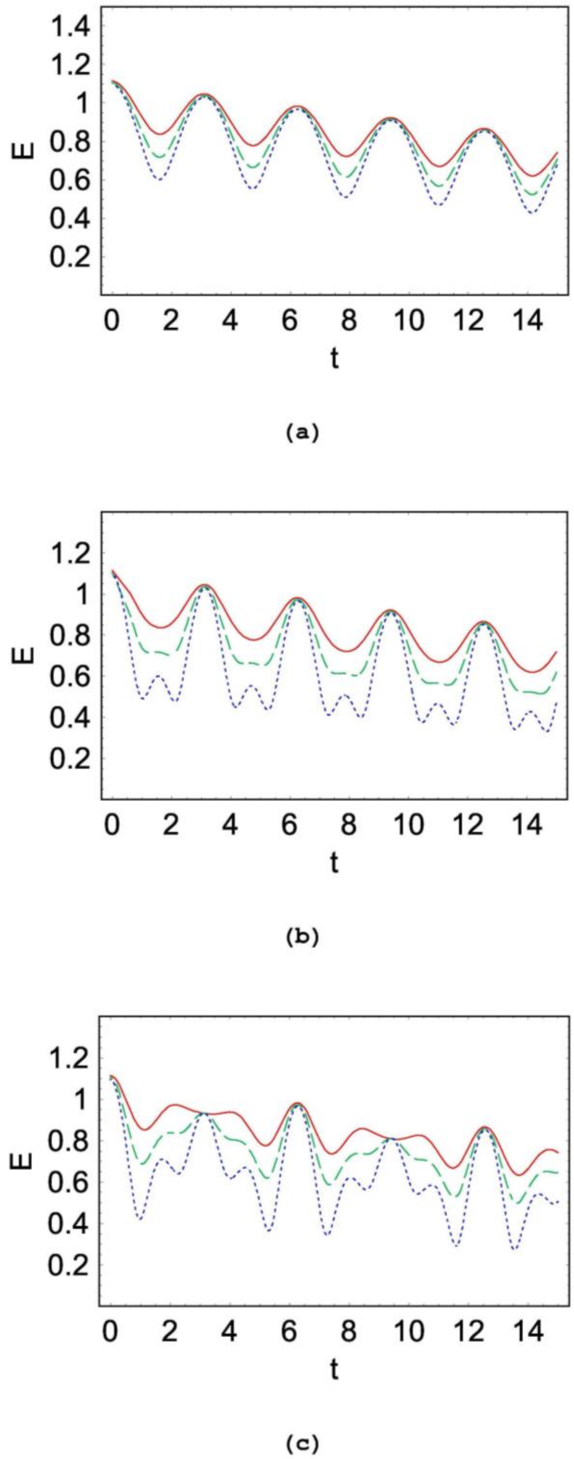

Figure 1
Time behavior of the energy given in Eq. (11) with the choice of $\xi(t)$ as Eq. (12). The value of ε is 0.1 for solid red line, 0.2 for long dashed green line, and 0.3 for short dashed blue line. The values of (β , ω_d) are (0, 1) for (a), (0.2, 1) for (b), and (0.2, 1.5) for (c). All other values are common and given by $\theta=\pi/2$, $\omega_r=1$, $\omega_p=2$, $Q=50$, $\xi_0=0.5$, $E_0=1$, and $C=1$.

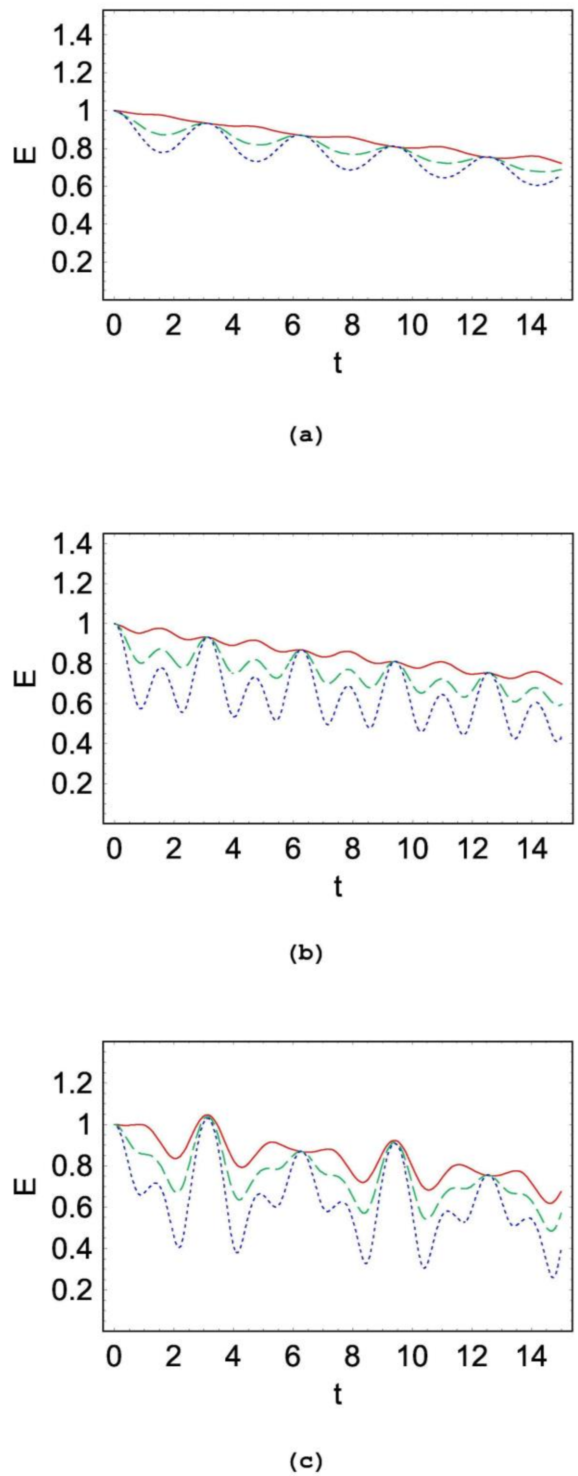

Figure 2
The same as Fig. 1, but $\theta = 0$.

$$J = \frac{2\pi e^{2\Gamma t}}{\Omega(t)}\left(E(t) + \frac{C\xi^2(t)}{2\Omega^2(t)}\right). \tag{10}$$

Because the change of the adiabatic invariant in time can be neglected in the adiabatic limit, it is possible to write $J(t) = J(0)$. Using this relation, we can see that the energy of the system can be represented in the form

$$E(t) = e^{-2\Gamma t}\frac{\Omega(t)}{\Omega(0)}\left(E(0) + \frac{C\xi^2(0)}{2\Omega^2(0)}\right) - \frac{C\xi^2(t)}{2\Omega^2(t)}. \tag{11}$$

For the case where the external force is given by

$$\xi(t) = \xi_0 \cos(\omega_d t + \theta), \tag{12}$$

where ξ_0, ω_d, and θ are constants, the time behavior of $E(t)$ is depicted in Figs. 1 and 2. Figure 1 corresponds to the case of maximum amplification which occurs at $\theta = \pi/2$, whereas Fig. 2 to greatest deamplification which occurs at $\theta = 0$. If we compare the three lines in Fig. 1 (or in Fig. 2), the fluctuation of the energy is high for large values of ε. Figures 1(a) and 2(a) correspond to the case that the parameter β is removed while others to the case that we consider it. We can confirm that the fluctuation of the energy is more or less complicated when β is not zero. Figures 1(c) and 2(c) are the case that the frequency ω_d is different from the resonance frequency ω_r, while others correspond to the case $\omega_d = \omega_r$. As the value of ω_d deviates from ω_r, the fluctuation of the energy becomes somewhat irregular.

4. SUMMARY AND CONCLUSION

We have investigated the properties of flux driven superconducting resonators which act like nanomechanical oscillators. This system can be used not only for the control of qubit states but also for their readout with high fidelity. A time-dependent Hamiltonian that describes the dynamics of magnetic flux in the superconducting resonator is derived from the equation for ϕ. The adiabatic invariant of the system is evaluated using the energy expression of the system. By making use of the characteristics of the adiabatic invariant, the time behavior of the energy of the resonator system is analyzed.

We have illustrated the response of the energy for the case that the external force is given by a sinusoidal form which is Eq. (12), under the assumption of the adiabatic change of the system. As amplification of the signal for qubit states proceeds, the scale of the energy has been fluctuated. We have confirmed that such fluctuation is high in case that the strength of the pumping is great. For the case $\beta \neq 0$, the response of the system is very complicated. If ω_d is different from the resonance frequency ω_r, the pattern of the energy variation is somewhat irregular. The analysis of superconducting resonator given in this work may provide a useful insight for the time evolution of the amplified signals of qubit states within the limit of adiabatic changes of parameters such as $\Omega(t)$.

DISCLOSURE STATEMENT

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