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Dynamics Analysis of a parametric amplification for driven Cooper-Pair Box Qubit Signals in Quantum computing system

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ABSTRACT

Electrically controllable Cooper-pair box qubits are important as particular resources for solid state quantum bit prototypes in quantum computing system. Because the detectable qubit signals of a Cooper-pair box is extremely weak, amplification of such signals using low-noise amplification techniques is necessary. Parametric amplification for driven Cooper-pair box qubit signals is investigated under a particular situation that effective stiffness of the spring of the parametric oscillator is increasing. The parametric amplification gain for the signal is derived and its time behavior is analyzed. The effects of amplification and deamplification of the signal are rigorously addressed on the basis of the illustration for the time behavior of the modulated signal. The phase condition for maximum amplification is also analyzed.

Keywords: Parametric Amplification, Cooper-Pair Box, Qubit, Gain

Abbreviations: CPB – Cooper-pair box

1. INTRODUCTION

Thanks to the rapid advancement of quantum computing technology, the construction of a scaled-up version of the quantum computing system composed of hundreds of qubits may be achievable in the near future. It is proved that quantum computers have the ability of efficient computing with extremely high-speed for specific computational tasks, using the principle of superposition and entanglement in quantum mechanics. A qubit, which is a basic element in quantum computing, is basically a two-level system. Among various implementations of qubit protocol, Cooper-pair box (CPB) quantum qubits [Tsai et al., 2001; Tsai et al., 2002] are an active research topic because they act as controllable versatile two-level quantum systems. Cooper-pair box is an artificial solid-state mesoscopic electronic circuit system composed of two superconducting islands which are weakly-linked to a superconducting reservoir via a Josephson junction [Armour et al., 2002; Houck et al., 2009]. An epoch-making advance to applying CPB charge qubits as a building block for quantum computation is recently achieved [Wallraff et al., 2005]. The quantized Cooper-pair charges in the box form a macroscopic quantum number state in a particular situation, leading to be a two-level quantum electronic system, or qubit. Two-level states of charges formed by a large number of electrons contained in a CPB are coherently superposed each other [Nakamura et al., 1999].

Though CPB qubits exhibit potential computational ability necessary for implementing quantum computation, the high-fidelity readout of their signals, for the purpose of processing and controlling real-time computing algorithms, requires high technology [Johnson, 2012]. A significant advance on optomechanical schemes concerning coherent control and efficient readout of quantum charge states of CPBs has been achieved by coupling the CPB qubit signals to a nanomechanical oscillator which acts as a resonator [Armour et al., 2002]. To interface extremely weak signals detected from the CPB to the macroscopic electronic devices which operate in noisy room temperature, a sensitive, noise-free amplifier is necessary. Parametric amplifier is commonly used for such purpose. The meaning of "parametric" is that at least one of the parameters in the resonant oscillator, such as frequency and/or damping, is modulated in time on the purpose of magnifying (pumping) the signal. As a consequence of such modulation, the amplitude of the readout resonant oscillator increases in time. The technique of parametric amplification in mechanical resonators is used not only for amplifying mechanical signals but also for enhancing the quality factor of the signals [Eichler et al., 2011].

We will consider parametric amplification of CPB signals coupled to a resonator through a time-varying modulation. The effects of linearly increasing effective stiffness on the amplification of the signal will be investigated. From an equation of motion for describing such parametric amplification, the solution for the time behavior of signal oscillation will be derived. The gain for the parametrically amplified signal amplitude will be rigorously evaluated and analyzed.

2. PARAMETRIC AMPLIFICATION OF A SIGNAL

Amplification of a signal is important for high precision readout of qubit signals from a CPB in quantum computing systems. A parametric amplifier enables us to enhance the level of a detected weak signal to a sufficiently magnified level so that it can be efficiently processed with macroscopic devices. We consider a parametric amplifier (parametric oscillator) of a CPB box signal that is described by the equation

$$m\ddot{q} + \frac{m\omega_0}{Q}\dot{q} + [k_0 + k_p(t)]q = F(t), \quad (1)$$

where k_0 is intrinsic spring constant of the oscillator $(k_0 = m\omega_0^2)$, $k_p(t)$ is a time-dependent perturbation in the stiffness of the spring, and $F(t)$ is an external driving force. We treat the case that $k_p(t)$ and $F(t)$ are given by

$$k_p(t) = k_1 t + k_2 \sin(2\omega_0 t), \quad (2)$$

$$F(t) = F_0 \cos(\omega_0 t + \phi), \quad (3)$$

where k_1, k_2 and F_0 are constants. We suppose that $k_1 t$ is a sufficiently slow varying function with the condition

$$k_1 << k_0\omega_0 . \quad (4)$$

If we think $k_0 + k_1 t$ as an effective stiffness of the spring for this system, it increases with time. This is the difference of our modulation of the signal from previous conventional modulations that suppose no alteration of the spring stiffness. Recently, the research for particular modulations of the signal through amplification, which are different from conventional modulations, have been carried out by several researchers. For instance, Dana et al studied parametric amplification for the case that the spring constant contains a nonlinear term [Dâna et al., 1998].

To find the solution of Eq. (1), we introduce a and its complex conjugate a^* as [Louisell, 1999; Rugar and Grütter, 1991]

$$a = \frac{dq}{dt} + i\omega_1^* q, \qquad a^* = \frac{dq}{dt} - i\omega_1 q, \quad (5)$$

where

$$\omega_1 = \omega_0\left(\sqrt{1 - \frac{1}{4Q^2}} + i\frac{1}{2Q}\right). \quad (6)$$

For convenience, we suppose the trial solution of $a(t)$ in the form

$$a(t) = A(t)e^{i\omega_0 t}. \quad (7)$$

Considering the condition for k_1 given in Eq. (4), we can assume that $dA(t)/dt \approx 0$. Then, the time derivative of $a(t)$ is given by

$$\frac{da(t)}{dt} = i\omega_1 a(t) + i\frac{a(t) - a^*(t)}{m(\omega_1 + \omega_1^*)}[k_1 t + k_2 \sin(2\omega_0 t)] + \frac{F_0}{m}\cos(\omega_0 t + \phi). \quad (8)$$

By inserting Eq. (7) into the above equation and using the high-Q approximation method represented in Ref. [Rugar and Grütter, 1991], we obtain

$$\left[i\left(\omega_1 - \omega_0 + \frac{k_1 t}{m}\right)A(t) - \frac{k_2}{2m(\omega_1 + \omega_1^*)}A^*(t) + \frac{F_0 e^{i\phi}}{2m}\right]e^{i\omega_0 t} = 0. \quad (9)$$

Using the approximation [Rugar and Grütter, 1991]

$$\omega_1 + \omega_1^* \approx 2\omega_0, \qquad \omega_1 - \omega_0 \approx \frac{i\omega_0}{2Q}, \quad (10)$$

Eq. (9) can be simplified to

$$\left(\frac{\omega_0}{2Q} - i\frac{k_1 t}{m}\right)A(t) + \frac{k_2}{4m\omega_0}A^*(t) - \frac{F_0 e^{i\phi}}{2m} = 0. \quad (11)$$

By solving this equation rigorously, we have

$$A(t) = A_R(t) + iA_I(t), \quad (12)$$

where

$$A_R(t) = \frac{F_0}{2mZ_-(t)}\left[\left(\frac{\omega_0}{2Q} - \frac{k_2}{4m\omega_0}\right)\cos\phi - \frac{k_1 t}{m}\sin\phi\right], \quad (13)$$

$$A_I(t) = \frac{F_0}{2mZ_-(t)}\left[\left(\frac{\omega_0}{2Q} + \frac{k_2}{4m\omega_0}\right)\sin\phi + \frac{k_1 t}{m}\cos\phi\right], \quad (14)$$

with

$$Z_\pm(t) = \frac{\omega_0^2}{4Q^2} \pm \frac{k_2^2}{16m^2\omega_0^2} + \frac{k_1^2 t^2}{m^2}. \quad (15)$$

Notice that $Z_{+}(t)$ in Eq. (15) will be used later.

Now we can express $q(t)$ as

$$q(t) = -i\frac{a - a^{*}}{\omega_1 + \omega_1^{*}} = q_0 \sin(\omega_0 t + \delta), \quad (16)$$

where q_0 is a time-dependent amplitude and δ is a phase, that are given by

$$q_0 = \frac{1}{\omega_0}\sqrt{A_R^2 + A_I^2}, \quad (17)$$

$$\delta = \tan^{-1}(A_I / A_R). \quad (18)$$

In the calculation of Eq. (17), we have used the first condition in Eq. (10).

The efficiency of the parametric amplifier can be estimated from the amplification gain which is the ratio of the amplitude of the amplified (pumped) signal to that of the unamplified one. Mathematically, the amplification gain can be evaluated from

$$G(\phi) = \frac{|q_0|_{pump\ on}}{|q_0|_{pump\ off}}. \quad (19)$$

Here, the situation of "pump off" corresponds to the case $k_p(t) = 0$. The use of Eq. (17) with Eqs. (13) and (14) as the value of $|q_0|_{pump\ on}$ and the use of the relation $|q_0|_{pump\ off} = F_0 Q/k_0$ yield

$$G(\phi) = \frac{\omega_0}{2QZ_{-}(t)}\left(Z_{+}(t) - \frac{k_2}{4Qm}\cos(2\phi) + \frac{k_1 k_2 t}{2m^2\omega_0}\sin(2\phi)\right)^{1/2}, \quad (20)$$

where Z_{+} is given in Eq. (15). For a particular case where $k_1 = 0$, this reduces to

$$G(\phi) = \left(\frac{\cos^2\phi}{[1 + k_2 Q/(2k_0)]^2} + \frac{\sin^2\phi}{[1 - k_2 Q/(2k_0)]^2}\right)^{1/2}, \quad (21)$$

which corresponds to the result of Ref. [Rugar and Grütter, 1991].

3. ANALYSIS OF THE AMPLIFICATION

From differentiation of Eq. (20) with respect to ϕ, we can find that maximum amplification occurs when the angle ϕ is $\phi_{MA} = (\pi - \Delta)/2$ where $\Delta = \tan^{-1}[2Qk_1 t/(m\omega_0)]$ and the maximum deamplification takes place when $\phi_{MDA} = -\Delta/2$. For $k_1 = 0$ or $t = 0$, we have $\phi_{MA} = \pi/2$ and $\phi_{MDA} = 0$ and these consequences agree with the results of Ref. [Rugar and Grütter, 1991].

The time evolution of $q(t)$ is shown in Figs. 1 and 2. Figure 1 corresponds to the case of amplification of the signal while Fig. 2 to deamplification of it. We can think that $k_0 + k_1 t$ is the effective stiffness of the spring, which increases with time. From Figs 1(b) and 1(c), the signal is amplified during some initial interval of time, but the amplification of the signal is more or less quenched abruptly after a finite time t. This phenomenon of quenching occurs earlier for the case of Fig. 2(c) than for the case of Fig. 2(b). This implies that the effective stiffness of the spring influences on the effect of the quenching. As the value of $k_1 t$ increases with time, such quenching happens early. By comparing Fig. 2(c) with Fig. 2(a) [or 2(b)], we can confirm that the signal rapidly deamplified as the effective stiffness of spring increases. The parametric amplification gain vs. t is illustrated in Fig. 3 for $\phi = (\pi - \Delta)/2$. These also have a peak at a finite time that corresponds to the moment in which the peak of oscillation amplitude occurs in Fig. 1.

4. CONCLUSION

Superpositions of distinct quantum states of a CPB can be detected by coupling the CPB qubit signals to a nanomechanical readout resonator. Because the readout signals are in general too weak to be used in nosy room temperature environment, they should be amplified to a sufficient level for the actual availability of them. The parametric amplification of qubit signals in CPB, that follow Eq. (1), is investigated from theoretical point of view. This system is a particular type of parametric amplifier that involves time-varying effective stiffness of the spring.

The time evolution of $q(t)$ and $G(\phi)$ are analyzed focusing on the effects of the constant k_1 on the system. For a sufficient large value of k_1t, the amplitude of oscillation cannot be maintained highly due to the increase of stiffness of the spring, leading to suppressing the amplitude of the readout signal oscillation. The effects of signal amplification are phase sensitive. Maximum amplification takes place when the phase satisfies $\phi = (\pi - \Delta)/2$ whereas greatest deamplification when $\phi = -\Delta/2$. To use the CPB qubit signals in quantum computation adequately, efficient readout of them through amplification is important.

DISCLOSURE STATEMENT

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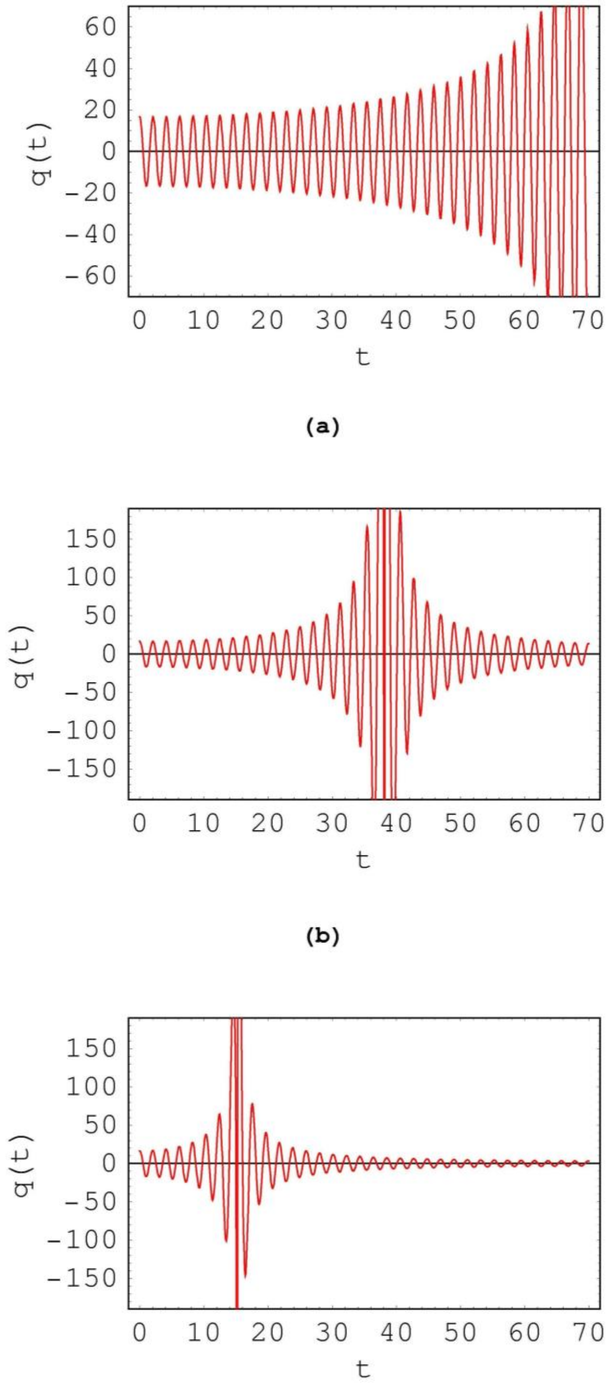

Figure 1

Amplification of the signal $q(t)$ in time where $\phi = (\pi - \Delta)/2$. The value of k_1 is 0.001 for (a), 0.002 for (b), and 0.005 for (c). We have used k_0=k_2=1, F_0=5, Q=5, and ω_0=3.

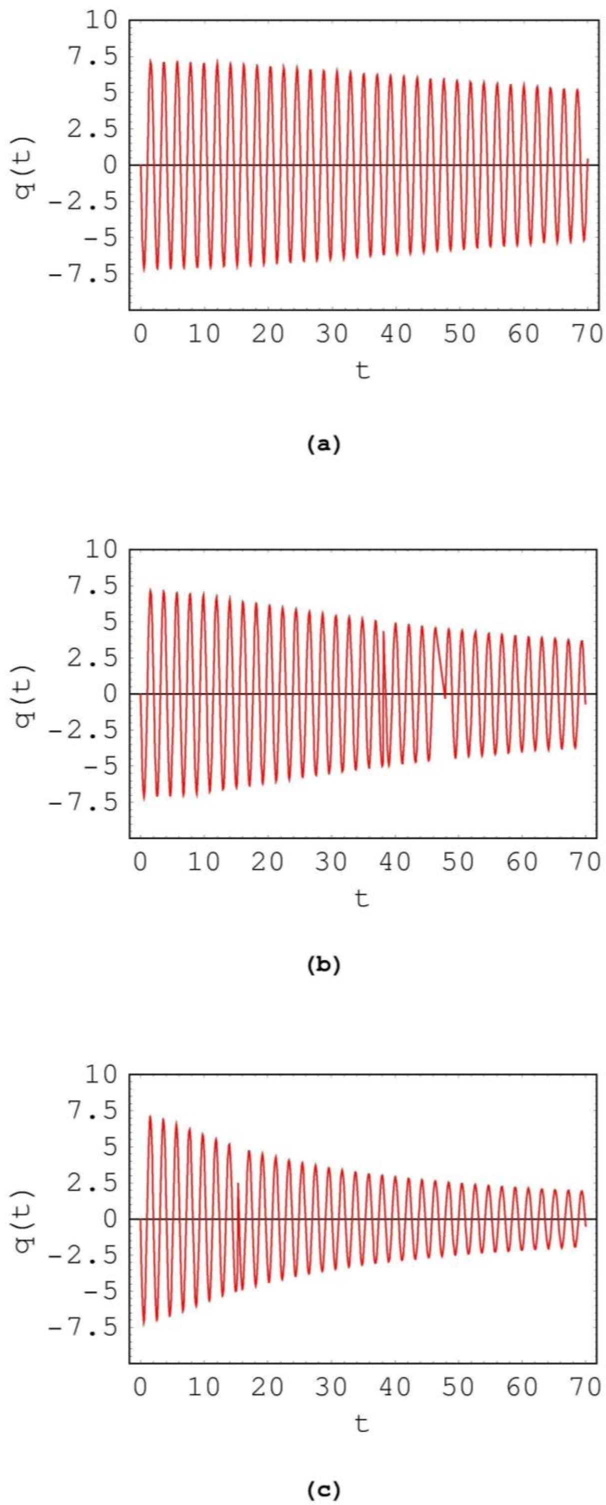

Figure 2
Deamplification of the signal $q(t)$ in time where $\phi = (\pi - \Delta)/2$. The value of k_1 is 0.001 for (a), 0.002 for (b), and 0.005 for (c). We have used k_0=k_2=1, F_0=5, Q=5, and ω_0=3.

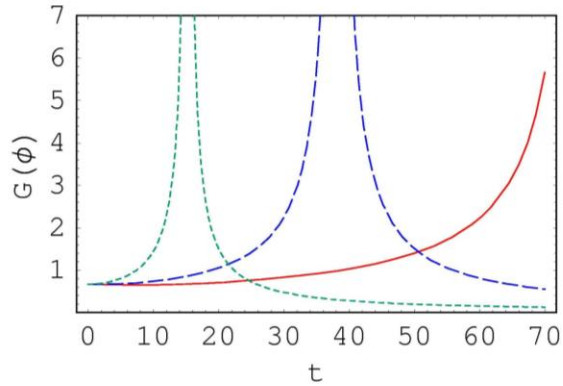

Figure 3
Amplification gain $G(\phi)$ in time where $\phi = (\pi - \Delta)/2$. The value of k_1 is 0.001 for solid red line, 0.002 for long dashed blue line, and 0.005 for short dashed green line. We have used k_0=k_2=1, F_0=5, Q=5, and ω_0=3.

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