Discovery, 1(3), September 1, 2012

RESEARCH

ISSN
2278–5469
EISSN
2278–5450

Discovery

# Worker scheduling problem using vertex coloring

Srividhya S[1]*, Balaji D[2]

1. Department of Mathematics, Hindustan College of Arts & Science, Chennai - 603103, Tamil Nadu, India
2. Department of Mathematics, Hindustan College of Arts & Science, Chennai - 603103, Tamil Nadu, India

☼Corresponding author: Srividhya S, Department of Mathematics, Hindustan College of Arts & Science, Chennai 603103, Tamil Nadu, India. E-Mail: srividhya2285@gmail.com

Publication History
Received: 18 July 2012
Accepted: 24 August 2012
Published: 1 September 2012

Citation
Srividhya S, Balaji D. Worker scheduling problem using vertex coloring. *Discovery*, 2012, 1(3), 48-50



## ABSTRACT
The objective of this paper is to provide simple way for solving worker scheduling problem in big industries. In any industry the worker has to satisfy their higher authorities. In many industries, the problem facing is worker scheduling problem. To overcome these problems we can schedule the timetable for workers by applying the concept of vertex coloring of graph theory.

Keywords: Points - vertices; Links - edges; I - Rows of a matrix; J - Columns of a matrix; Chromatic number - The *least* number of colors required to do a coloring of a graph.

Abbreviations: G - graph; X - Group of workers; w - workers; A (G) - adjacency matrix

# 1. INTRIODUCTION

Vertex coloring models to a number of scheduling problems. In the cleanest form, a given set of jobs need to be assigned to time slots, each job requires one such slot. Jobs can be scheduled in any order, but pairs of jobs may be in conflict in the sense that they may not be assigned to the same time slot, for example because they both rely on a shared resource. The corresponding graph contains a vertex for every job and an edge for every conflicting pair of jobs. The chromatic number of the graph is exactly the minimum number, the optimal time to finish all jobs without conflicts (Bondy & Murty, 1976). Details of the scheduling problem define the structure of the graph. For example, when assigning aircraft to flights, the resulting conflict graph is an interval graph, so the coloring problem can be solved efficiently. In bandwidth allocation to radio stations, the resulting conflict graph is a unit disk graph, so the coloring problem is 3-approximable. In any organisation like Industries, hospitals, transport department which are working continuously, used to divide their works in a shift basis to minimize the complexity of the work and to maximize their profit. This scheduling problem consists of assigning a schedule to each worker for around 8hours a day. According to Wren, "Scheduling is the arrangement of objects into pattern in time or space in such a way that some goals are achieved or nearly achieved". Rostering means, placing of resources into slots in a pattern. A big manufacturing company is one of the examples for the organisation at which workers are scheduling into shift basis. In an industry the higher authority must assign works to the workers in shifts to meet the demand of the product. In this paper we mainly considered about the workers shifts which are given for worker named as worker roster. The industry has to provide 24hours worker coverage at some levels to manufacture the products. In this paper we consider the problem involving the coloring of vertices in a graph theory and we have an algorithm for vertex coloring. To find the easy way for scheduling shifts to workers we used vertex coloring (Aldous & Wilson, 2000).

## 1.1. Definition for vertex coloring

Let G be a simple graph. A k-coloring of G is an assignment of at most k-colors to the vertices of G in such a way that adjacent vertices are assigned different colors. If G has k-coloring, then G is k-colorable. The chromatic number of a graph is the *least* number of colors required to do a coloring of a graph. The chromatic number for the graph G, denoted by X (G) is the smallest number k for which G is k-colorable.

Example:

The Fig.1 shows it has 6 vertices and 12 edges. Minimum colors used in the given graph are Blue, Green, and Red i.e. 3-colors. Therefore chromatic number for this graph is X (G) = 3

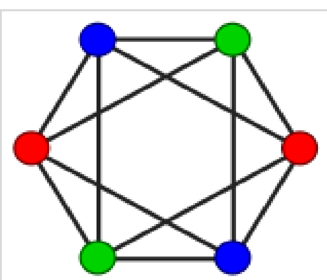

**Figure 1**
The chromatic number of a graph

A proper k-coloring is a partition of the vertices V= X1+X2+X3+..........+Xk such that each Xi (i=1,2,3,..) is a set consists of both senior and junior workers. To schedule the shifts using vertex coloring, let us split the set of workers into four disjoint subsets corresponding to their experience and we are not splitting the workers like experienced separately, fresher's separately because if any problem arises the experienced staff are needed to rectify their mistakes. So we are keeping the disjoint sets of having both experienced and freshers. The management has to frame a schedule of the shift types in order to make them comfortable. Workers can work in three shifts because the industry is very big and have to satisfy the customers.

Morning shift : 6am-2pm
Evening shift : 2pm-10pm
Night shift : 10pm-6pm

Let us consider that there are 30 workers working in an industry. Any senior staff has to frame the timetable in three shifts as like mentioned above. The senior person has to be more careful in the following constraints:

- Any worker doesn't work the morning shift, evening shift and night shift on the same day.
- A worker may take a holiday on any day
- A worker doesn't work the next day after a night shift
- Assign equal shift type per week
- Has to make balanced workload among the workers.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 2 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 4 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 5 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 6 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 9 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 10 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 12 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 14 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 15 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 16 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 17 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 18 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 19 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 20 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 21 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 23 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 24 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 25 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 26 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 27 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 28 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 29 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 30 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |

**Figure 2**
Adjacency matrix for the workers

# 2. APPLYING THE CONCEPT OF GRAPH THEORY IN WORKER SCHEDULING PROBLEM

Graph theory is very much used in scheduling problem like Nurse scheduling problem, drivers scheduling problem etc. Vertex coloring concept in graph theory is useful in allocating the resources like man, machine, and vehicles in a correct way without having any conflicts. In this paper we are considering the constraints, workers with different stages of experience can be in same shift so that if any problem occurs the senior most workers may correct or avoid the problem and also to finish their works in on time. To solve this scheduling problem by using graph theory, let us consider that there are 30 workers working in an industry. Any senior staff has to frame the timetable in three shifts as mentioned above. Considering 8 senior most staff were divided and put them in each group say X1, X2, X3.

The senior most staff is dividing the group of workers in the following way and also with experienced staff in each group. The workers are named as w1, w2, w3, w4, w5 ...w30. They are dividing the group which consists of 10 workers. By the definition of proper k-coloring given above, we can split the set of workers into 3 disjoint subsets corresponding to the three areas.

The first group X1 = {w1, w5, w13, w18, w19, w22, w27, w28, w29, w30}
Second group X2 = {w2, w4, w6, w7, w21, w23, w24, w16, w12, w10}
Third group X3 = {w3, w8, w9, w11, w14, w15, w17, w20, w25, w26}

By using the above sets a matrix was created for the workers. It was 30 X 30 matrixes. In that matrix, workers names are taken as i and j. Then ij th entry is put according to the above sets.

If any two workers are in same group, then ij th entry = {1
{0 otherwise.

Adjacency matrix for the worker set: (for our convenience let us take w1, w2, w3...w30 as simply 1, 2, 3, 4...30)

## 2.1. Definition for adjacency matrix

Let G be a graph with V (G) = {1, 2, 3......n} and E (G) = {$e_1$, $e_2$, $e_3$....... em} the adjacency matrix of G denoted by A (G) is the n x n matrix defined as follows. The rows and the columns of A (G) are indexed by V (G).

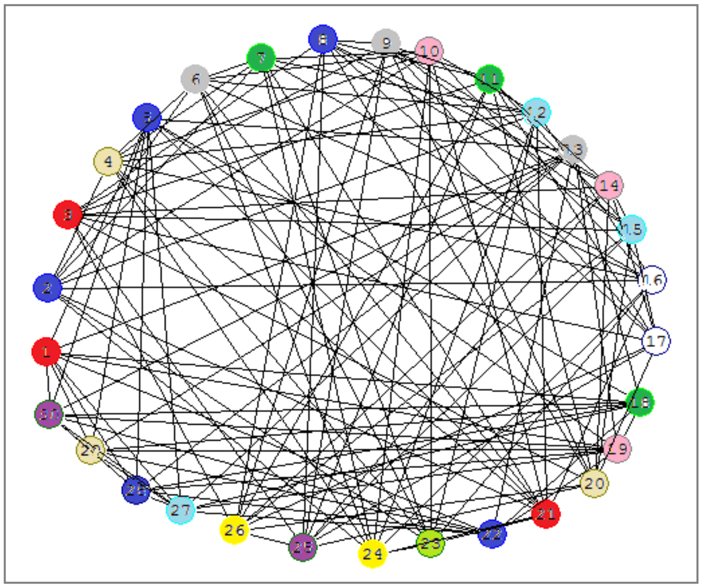

**Figure 3**
Graph theory using the software 'Grin'

If i ≠ j then the (i, j) th entry is 1 for i and j are adjacent. The (i, j) entry of A (G) is 0 for i = 1, 2, 3…….n. we can denote A (G) as A. The following matrix is the adjacency matrix for the set of workers which shows that who can work together in the same shift. i.e. in the adjacency matrix 1 – denotes that we can connect those points (vertices).

0 – denotes that we cannot connect those points

Using the matrix from Fig.2, a graph is constructed by taking workers as vertices (points). If two workers are in the same group according to the above disjoint sets X1, X2, X3 the corresponding vertices are combined using edges. In a graph theory the above matrix can be named as adjacency matrix. It is a m x m matrix whose ij-th element is the number of edges joining vertex i and vertex j. In the next part vertices in graph were colored. By the definition of the vertex coloring in graph theory which is given above, the adjacent vertices can be colored using different colors.

## 3. GREEDY ALGORITHM FOR VERTEX COLORING

Greedy Algorithm is the most popular algorithm for vertex coloring in graph theory.

### 3.1. Algorithm

Start with a graph G and list of colors say 1, 2, 3, 4……

### 3.2. Step 1

Label the vertices say v1, v2, v3….. in any manner.

### 3.3. Step 2

Identify the uncolored vertex labelled with the earliest letter in the vertices like w1, w2, w3…..color it with the first colour in the list not used for any adjacent colored vertex. Repeat step 2 until all the vertices are colored, and then stop.

### 3.4. Step 3

A vertex coloring of G has been obtained. The number of colors used depends on the labelling chosen for the vertices in step 1. By using the greedy algorithm we have drawn the graph and colored the vertices based on the definition of the vertex coloring. After drawing the graph, adjacent vertices were colored with different colors. The following details shows that the colors are used for the vertices (points) based on greedy algorithm.

Point 1 2 3 4 5 6 7 8 9 10
Color 1 2 1 8 2 3 4 2 3 5
Point 11 12 13 14 15 16 17 18 19 20
Color 4 6 3 5 6 7 7 4 5 8
Point 21 22 23 24 25 26 27 28 29 30
Color 1 2 9 10 9 10 6 7 8 9
NetWork : wsp grin.next
Type : undirNet
Number of Points : 30
Number of Edges : 135
Chromatic Number = 10

By using the software of graph theory called 'Grin' we got the final colored graph. The resulting graph is shown in Fig.3. Table 1 shows the group of workers details after applying vertex coloring techniques. According to Table 1, workers w1 and w27 can be in the same shift and can work together because w1 and w27 are the members of the same group X1. But w1 and w2 cannot work together because they are in different groups i.e. w1 € X1 and w2 € X2. More than one group can work together in the same shift but the workers must be in the same group otherwise some constraints may be violated.

**Table 1** Group of workers details after applying vertex coloring techniques

| Group | Names of the workers |
|---|---|
| **X1** | **w1, w5, w13, w18, w19, w22, w27, w28, w29, w30** |
| **X2** | **w2, w4 , w6, w7, w21, w23, w24, w16, w12, w10** |
| **X3** | **w3, w8, w9, w11, w14, w15, w17, w20, w25, w26** |

## 4. CONCLUSION

In this paper we have applied the concept of vertex coloring to schedule workers which are facing by continuously working organization. Here, we have divided the workers into three groups using the definition of proper k-coloring and using the sets we have formed the adjacency matrix using adjacency we have drawn the graph and colored. According to the above table workers w1 and w27 can be in the same shift and can work together because w1 and w27 are the members of the same group X1. But w1 and w2 cannot work together because they are in different groups i.e w1 € X1 and w2 € X2. More than one group can work together in the same shift but the workers must be in the same group otherwise some constraints may be violated. In any scheduling problem vertex coloring of graph theory can be applied to solve the scheduling problem in an organization.

## DISCLOSURE STATEMENT

There is no financial support for the proposed research work.

## ACKNOWLEDGEMENT

We extend our special thanks to our college, principal, colleagues, friends and family members for their moral support.

## REFERENCES

1. Bondy JA, Murty USR. Graph theory with applications. *The Macmillan Press Ltd.*, 1976
2. Joan M Aldous, Robin J Wilson. Graphs and applications, *An Introductory Approach*. 2000, 1, pp: 444